Source Side Information Can Increase the Joint Source-Channel Coding Error Exponent*

Yangfan Zhong, Fady Alajaji and L. Lorne Campbell
Department of Mathematics and Statistics
Queen's University, Kingston, ON K7L 3N6, Canada
Email: {yangfan,fady,campblll}@mast.queensu.ca

***Abstract*— We study the joint source-channel coding (JSCC) error exponent for discrete memoryless source-channel systems with side information which is correlated to the transmitted source. Two cases are considered: (1) the side information is available only at the decoder; (2) the side information is available at both the encoder and decoder. We employ the method of types to establish a lower bound for the JSCC error exponent for each case. As a consequence, a JSCC theorem on the reliable transmissibility of the source over the channel is obtained. It is noted that the same JSCC theorem applies for both cases. For binary sources and symmetric channels, we derive a sufficient condition for which the side information at the decoder can strictly improve the JSCC error exponent. Numerical results show that side information can enlarge the region for reliable transmissibility and increase the JSCC error exponent for a wide class of source-channel parameters.**

I. Introduction

In [2], Csiszár establishes a lower bound and an upper bound for the lossless (fixed-length) joint source-channel coding (JSCC) error exponent for systems consisting of a discrete memoryless source (DMS) and a discrete memoryless channel (DMC). Given a DMS $\{Q_S : \mathcal{S}\}$, a DMC $\{W_{Y|X} : \mathcal{X} \to \mathcal{Y}\}$, and a transmission rate t ($t > 0$ is a constant measured in source symbol/channel use), he proved that the JSCC error exponent $E_J(Q_S, W_{Y|X}, t)$ satisfies

$$\underline{E}_J(Q_S, W_{Y|X}, t) \le E_J(Q_S, W_{Y|X}, t) \le \overline{E}_J(Q_S, W_{Y|X}, t),$$

where

$$\underline{E}_J(Q_S, W_{Y|X}, t) \triangleq \min_{P_S}[tD(P_S\|Q_S) + E_r(tH_{P_S}(S), W_{Y|X})] \quad (1)$$

is called Csiszár's random-coding lower bound to E_J, and

$$\overline{E}_J(Q_S, W_{Y|X}, t) \triangleq \min_{P_S}[tD(P_S\|Q_S) + E_{sp}(tH_{P_S}(S), W_{Y|X})] \quad (2)$$

is called Csiszár's sphere-packing upper bound to E_J. In (1) and (2), $D(P_S\|Q_S)$ is the Kullback-leibler divergence, and $E_r(R, W_{Y|X})$ and $E_{sp}(R, W_{Y|X})$ are the random-coding lower bound and the sphere-packing upper bound, respectively, for the channel error exponent $E(R, W_{Y|X})$, i.e., $E_r(R, W_{Y|X}) \le E(R, W_{Y|X}) \le E_{sp}(R, W_{Y|X})$. The computation of the bounds for E_J and the sufficient and necessary condition for which $\underline{E}_J = \overline{E}_J$ have been studied in [5].

*This work was supported in part by NSERC of Canada.

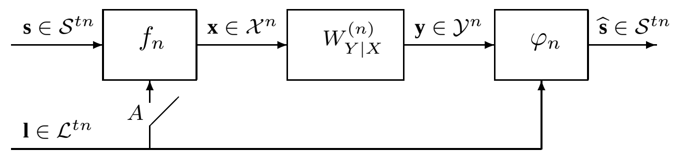

Fig. 1. The JSCC system with source side information.

In this work we extend Csiszár's JSCC problem by considering the availability of side information on the transmitted source at the decoder or at both the decoder and the encoder. Our system is depicted in Fig. 1. The source message pair $(\mathbf{s}, \mathbf{l})$ of length tn is drawn in an independent and identically distributed (i.i.d.) manner from a joint distribution $\{Q_{SL} : \mathcal{S} \times \mathcal{L}\}$. We need to transmit the source message $\mathbf{s}$ over the DMC $W_{Y|X}$ via JSCC block codes of length n and transmission rate t. The source message $\mathbf{l}$, viewed as a noisy observation of $\mathbf{s}$, contains the source side information and helps the decoder reconstruct $\mathbf{s}$. Now consider the following two cases:

1) Switch A is open, i.e., the side information is only available at the decoder;
2) Switch A is closed, i.e., the side information is available at both the encoder and the decoder.

We establish an (achievable) lower bound for the JSCC error exponent for each case. The sufficient and necessary condition for which the source[1] Q_S can be reliably transmitted over the channel $W_{Y|X}$, i.e., the JSCC theorem, is also formulated for each case. It is seen that the same JSCC theorem applies for both cases. A sufficient condition for which the source side information at the decoder can strictly enlarge the JSCC error exponent for a system consisting of a binary source and a symmetric channel is derived. Numerical results show that side information (at the decoder) not only enlarges the region of the source-channel parameters for which reliable transmissibility is possible, but it can also provide a noticeable increase in the JSCC error exponent for a large class of source-channel parameters.

[1]We refer to a single source by using its distribution. Here Q_S denotes the marginal distribution of Q_{SL} on $\mathcal{S}$.

II. Problem Formulation

For any finite set (or alphabet) $\mathcal{X}$, the size of $\mathcal{X}$ is denoted by $|\mathcal{X}|$. For any finite sets $\mathcal{X}$ and $\mathcal{Y}$, the set of all probability distributions on $\mathcal{X}$ is denoted by $\mathcal{P}(\mathcal{X})$, and the set of all conditional distributions $V_{Y|X}:\mathcal{X}\to\mathcal{Y}$ is denoted by $\mathcal{P}(\mathcal{Y}|\mathcal{X})$. For finite sets $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ with joint distribution $P_{XYZ}\in\mathcal{P}(\mathcal{X}\times\mathcal{Y}\times\mathcal{Z})$, we use P_X, P_{XY}, $P_{YZ|X}$, etc, to denote the corresponding marginal and conditional probabilities induced from P_{XYZ}. For any distribution $P_{XYZ}\in\mathcal{P}(\mathcal{X}\times\mathcal{Y}\times\mathcal{Z})$, $H_{P_{XYZ}}(\cdot)$ and $I_{P_{XYZ}}(\cdot;\cdot)$ denote the entropy and mutual information under P_{XYZ}, respectively. $D(P_X \parallel Q_X)$ denotes the Kullback-Leibler divergence between distributions $P_X\in\mathcal{P}(\mathcal{X})$ and $Q_X\in\mathcal{P}(\mathcal{X})$. $D(V_{Y|X}\parallel W_{Y|X}|P_X)$ denotes the Kullback-Leibler divergence between conditional distributions $V_{Y|X}\in\mathcal{P}(\mathcal{Y}|\mathcal{X})$ and $W_{Y|X}\in\mathcal{P}(\mathcal{Y}|\mathcal{X})$ conditional on distribution $P_X\in\mathcal{P}(\mathcal{X})$. Given distributions $P_X\in\mathcal{P}(\mathcal{X})$ and $W_{Y|X}\in\mathcal{P}(\mathcal{Y}|\mathcal{X})$, let $P_X^{(n)}$ and $W_{Y|X}^{(n)}$ be their n-dimensional product distributions, respectively. All logarithms and exponentials in this paper are in base 2; furthermore, all alphabets are finite.

We consider a communication system consisting of two correlated DMS's $\{Q_{SL}:\mathcal{S}\times\mathcal{L}\}$ with finite alphabet $\mathcal{S}\times\mathcal{L}$ and joint distribution Q_{SL}, and a DMC $\{W_{Y|X}:\mathcal{X}\to\mathcal{Y}\}$ with finite input alphabet $\mathcal{X}$, finite output alphabet $\mathcal{Y}$, and transition probability distribution $W_{Y|X}$. We need to transmit the source Q_S over the channel $W_{Y|X}$ with side information Q_L available at (1) the decoder only, and (2) both the encoder and the decoder. We next define the probability of error and the JSCC error exponent for each case.

1) A joint source-channel (JSC) code of block length n and transmission rate $t>0$ for the system with side information at the decoder is a pair of mappings, (f_n,φ_n), where $f_n:\mathcal{S}^{tn}\to\mathcal{X}^n$ is the encoder, and $\psi_n:\mathcal{Y}^n\times\mathcal{L}^{tn}\to\mathcal{S}^{tn}$ is the decoder. The probability of error is given by

$$P_{e,n}^{SID}(Q_{SL},W_{Y|X},t) \triangleq \sum_{(\mathbf{s},\mathbf{l})\in\mathcal{S}^{tn}\times\mathcal{L}^{tn}} Q_{SL}^{(tn)}(\mathbf{s},\mathbf{l}) \sum_{\mathbf{y}:\varphi_n(\mathbf{y},\mathbf{l})\neq\mathbf{s}} W_{Y|X}^{(n)}(\mathbf{y}|f_n(\mathbf{s})).$$

Given Q_{SL}, $W_{Y|X}$ and $t>0$, the JSCC error exponent $E_J^{SID}(Q_{SL},W_{Y|X},t)$ is defined as supremum of the set of all numbers E for which there exists a sequence of JSC codes (f_n,φ_n) with blocklength n and transmission rate t such that

$$E\le\liminf_{n\to\infty}-\frac{1}{n}\log_2 P_{e,n}^{SID}(Q_{SL},W_{Y|X},t). \quad (3)$$

2) A JSC code of block length n and transmission rate $t>0$ for the system with side information at both the encoder and the decoder is a pair of mappings, (f_n,φ_n), where $f_n:\mathcal{S}^{tn}\times\mathcal{L}^{tn}\to\mathcal{X}^n$ is the encoder, and $\psi_n:\mathcal{Y}^n\times\mathcal{L}^{tn}\to\mathcal{S}^{tn}$ is the decoder. The probability of error is given by

$$P_{e,n}^{SIED}(Q_{SL},W_{Y|X},t) \triangleq \sum_{(\mathbf{s},\mathbf{l})\in\mathcal{S}^{tn}\times\mathcal{L}^{tn}} Q_{SL}^{(tn)}(\mathbf{s},\mathbf{l}) \sum_{\mathbf{y}:\varphi_n(\mathbf{y},\mathbf{l})\neq\mathbf{s}} W_{Y|X}^{(n)}(\mathbf{y}|f_n(\mathbf{s},\mathbf{l})).$$

Given Q_{SL}, $W_{Y|X}$ and $t>0$, the JSCC error exponent $E_J^{SIED}(Q_{SL},W_{Y|X},t)$ is defined as supremum of the set of all numbers E for which there exists a sequence of JSC codes (f_n,φ_n) with blocklength n and transmission rate t such that

$$E\le\liminf_{n\to\infty}-\frac{1}{n}\log_2 P_{e,n}^{SIED}(Q_{SL},W_{Y|X},t). \quad (4)$$

In this paper we shall lower bound the JSCC error exponent for each case.

III. Main Results

Given $W_{Y|X}\in\mathcal{P}(\mathcal{Y}|\mathcal{X})$ and any $R>0$, define the random-coding exponent for $W_{Y|X}$ by

$$E_r(R,W_{Y|X})\triangleq\max_{P_X\in\mathcal{P}(\mathcal{X})}E_r(R,W_{Y|X},P_X)$$

where

$$E_r(R,W_{Y|X},P_X) \triangleq \min_{V_{Y|X}\in\mathcal{P}(\mathcal{Y}|\mathcal{X})}\Big[D(V_{Y|X}\parallel W_{Y|X}|P_X) + \left|I_{P_XV_{Y|X}}(X;Y)-R\right|^+\Big],$$

where $|x|^+=\max\{0,x\}$. For the joint distribution of the sources Q_{SL}, we can look at the conditional distribution $Q_{L|S}\in\mathcal{P}(\mathcal{L}|\mathcal{S})$ as a dummy channel between Q_S and Q_L. For any $P_S\in\mathcal{P}(\mathcal{S})$ and any $R\le H_{P_S}(S)$, we define an exponent for the dummy channel $Q_{L|S}$ by

$$\begin{aligned} e_r(R,Q_{L|S},P_S) &\triangleq E_r(H_{P_S}(S)-R,Q_{L|S},P_S)\\ &= \min_{P_{L|S}\in\mathcal{P}(\mathcal{L}|\mathcal{S})}\Big[D(P_{L|S}\|Q_{L|S}|P_S) + |R-H_{P_SP_{L|S}}(S|L)|^+\Big]. \end{aligned}$$

Note that $E_r(R,W_{Y|U})$ is a strictly decreasing function of R and vanishes at the channel capacity of $W_{Y|X}$, $C(W_{Y|X})=\max_{P_X}I_{P_XW_{Y|X}}(X;Y)$. Accordingly, $e_r(R,Q_{L|S},P_S)$ is a strictly increasing function of R and is zero if and only if $R\le H_{P_SQ_{L|S}}(S|L)$. Let

$$E_{SI}(P_S,Q_{L|S},W_{Y|X})\triangleq\max_{0\le R\le tH_{P_S}(S)} \min\left\{E_r(R,W_{Y|X}),te_r\left(\frac{R}{t},Q_{L|S},P_S\right)\right\}. \quad (5)$$

Theorem 1: Given Q_{SL}, $W_{Y|X}$ and $t>0$, when the side information Q_L is available only at the decoder, the JSCC error exponent satisfies

$$\begin{aligned} E_J^{SID}(Q_{SL},W_{Y|X},t) &\ge \underline{E}_J^{SID}(Q_{SL},W_{Y|X},t)\\ &\triangleq \min_{P_S\in\mathcal{P}(\mathcal{S})}\left[tD(P_S\|Q_S)+E_r^*(P_S,W_{Y|X})\right], \quad (6)\end{aligned}$$

where

$$E_r^*(P_S,W_{Y|X}) = \max\left\{E_r(tH_{P_S}(S),W_{Y|U}),E_{SI}(P_S,Q_{L|S},W_{Y|U})\right\}.$$

The proof of Theorem 1 follows from a two-stage encoding two-stage decoding scheme which combines the approaches of Csiszár [2] and Oohama and Han [4] and is based on the method of types. In particular, at the decoding side, we employ a generalized maximum mutual information decoder followed by a minimum conditional entropy decoder.

Given $W_{Y|X}$ and an arbitrary finite alphabet $\mathcal{U}$, we introduce an auxiliary RV $U \in \mathcal{U}$ such that U, X, and Y form a Markov chain in this order, $U \to X \to Y$, i.e., the conditional probability $\Pr(Y = y|U = u, X = x) = W_{Y|X}(y|x)$ for any $u \in \mathcal{U}$, $x \in \mathcal{X}$ and $y \in \mathcal{Y}$. For the sake of convenience, we denote the conditional probability $\Pr(Y = y|U = u, X = x)$ induced from $W_{Y|X}$ by $W_{Y|UX}$, and call it the augmented channel of $W_{Y|X}$.

For arbitrary and finite alphabet $\mathcal{U}$, distribution $P_U \in \mathcal{P}(\mathcal{U})$, and every $R > 0$, define

$$E_r^{\dagger}(R, P_U, W_{Y|X}) = \max_{P_{X|U} \in \mathcal{P}(\mathcal{X}|\mathcal{U})} E_r^{\dagger}(R, W_{Y|UX}, P_U, P_{X|U}) \tag{7}$$

where

$$\begin{aligned} &E_r^{\dagger}(R, W_{Y|UX}, P_U, P_{X|U}) \\ &\triangleq \min_{V_{Y|UX} \in \mathcal{P}(\mathcal{Y}|\mathcal{U}\times\mathcal{X})} \big[D(V_{Y|UX} \parallel W_{Y|UX}|P_{UX}) \\ &\quad + \left|I_{P_{UX}V_{Y|UX}}(X;Y|U) - R\right|^{+}\big], \end{aligned} \tag{8}$$

where $P_{UX} = P_U P_{X|U}$.

Theorem 2: Given Q_{SL}, $W_{Y|X}$ and $t > 0$, when the side information Q_L is available at both the encoder and the decoder, and for arbitrary and finite alphabet $\mathcal{U}$ and distribution $P_U \in \mathcal{P}(\mathcal{U})$, the JSCC error exponent satisfies

$$\begin{aligned} E_J^{SIED}(Q_{SL}, W_{Y|X}, t) \ \ge\ & \min_{P_{SL} \in \mathcal{P}(\mathcal{S}\times\mathcal{L})} [tD(P_{SL}\|Q_{SL}) \\ & + E_r^{\dagger}\left(tH_{P_{SL}}(S|L), P_U, W_{Y|X}\right)] . \end{aligned}$$

The proof of the above lower bound is also based on the method of types, employing a pre-encoding function at both the encoder and the decoder on the side information message **l**, and a generalized maximum conditional mutual information decoder.

Of course, we can maximize the above lower bound by taking the supremum over all finite alphabets $\mathcal{U}$ and taking the maximum over all possible distributions $P_U \in \mathcal{P}(\mathcal{U})$, i.e.,

$$\begin{aligned} &E_J^{SIED}(Q_{SL}, W_{Y|X}, t) \ge \underline{E}_J^{SIED}(Q_{SL}, W_{Y|X}, t) \\ &\triangleq \sup_{\mathcal{U}} \max_{P_U} \min_{P_{SL} \in \mathcal{P}(\mathcal{S}\times\mathcal{L})} [tD(P_{SL}\|Q_{SL}) \\ &\quad + E_r^{\dagger}\left(tH_{P_{SL}}(S|L), P_U, W_{Y|X}\right)] . \end{aligned} \tag{9}$$

By examining the positivity of the lower bound $\underline{E}_J^{SID}$ or $\underline{E}_J^{SIED}$, we obtain a sufficient condition for which the source Q_S can be reliably transmitted over the channel. We also can prove a necessary condition by using Fano's inequality [1], and thus complete the JSCC theorem. It turns out that the availability of the side information at the encoder does not make any difference on the transmissibility of the source Q_S over the channel $W_{Y|X}$ if the side information Q_L is already known by the decoder.

Theorem 3: (JSCC Theorem) Given $Q_{SL}, W_{Y|X}$ and $t > 0$, when the side information Q_L is available either (1) at only the decoder, or (2) at both the encoder and the decoder, we have the following conditions.

1) The source Q_S can be transmitted over the channel $W_{Y|X}$ with an arbitrarily small probability of error if $tH_{Q_{SL}}(S|L) < C(W_{Y|X})$, where $C(W_{Y|X})$ is the channel capacity of $W_{Y|X}$.
2) Conversely, if the source Q_S can be transmitted over the channel $W_{Y|X}$ with an arbitrarily small probability of error, then $tH_{Q_{SL}}(S|L) \le C(W_{Y|X})$.

IV. Discussion

By definition, we know that the error exponent with side information must be larger than the one without side information, and the error exponent with side information at both the encoder and decoder must be larger than the one with side information only at the decoder. In other words, any lower bound of E_J must be a lower bound of E_J^{SID} and E_J^{SIED}, and similarly, the lower bound of E_J^{SID} is automatically a lower bound of E_J^{SIED}. Furthermore, we can relate the derived lower bounds as follows.

Theorem 4:

$$\begin{aligned} \underline{E}_J(Q_{SL}, W_{Y|X}, t) &\le \underline{E}_J^{SID}(Q_{SL}, W_{Y|X}, t) \\ &\le \underline{E}_J^{SIED}(Q_{SL}, W_{Y|X}, t). \end{aligned} \tag{10}$$

We next observe that the side information does not only enlarge the achievable region for transmission (see Theorem 3 and recall that $H_{Q_{SL}}(S|L) \le H_{Q_S}(S)$), but also improves the reliability of transmission. Obviously, if the sources Q_{SL} and the channel $W_{Y|X}$ satisfy $tH_{Q_{SL}}(S|L) < C(W_{Y|X}) < tH_{Q_S}(S)$, then we have

$$\begin{aligned} E_J^{SID}(Q_{SL}, W_{Y|X}, t) &\ge \underline{E}_J^{SID}(Q_{SL}, W_{Y|X}, t) \\ &> 0 = E_J(Q_{SL}, W_{Y|X}, t). \end{aligned}$$

Recalling that we also have an upper bound for E_J given by (2), thus, we can study the benefits of E_J^{SID} over E_J by comparing the lower bound $\underline{E}_J^{SID}$ with the upper bound $\overline{E}_J$.

For a DMS Q_S, denote Gallager's source function by

$$E_s(\rho, Q_S) \triangleq (1+\rho)\log_2 \sum_{s\in\mathcal{S}} Q_S(s)^{\frac{1}{1+\rho}}. \tag{11}$$

For a DMC $W_{Y|X}$, denote Gallager's channel function by

$$E_o(\rho, W_{Y|X}) \triangleq \max_{P_X \in \mathcal{P}(\mathcal{X})} E_o(\rho, P_X, W_{Y|X}) \tag{12}$$

where

$$E_o(\rho, P_X, W_{Y|X}) \triangleq -\log \sum_{y\in\mathcal{Y}} \left(\sum_{x\in\mathcal{X}} P_X(x) W_{Y|X}^{\frac{1}{1+\rho}}(y|x) \right)^{1+\rho}, \tag{13}$$

$\rho \ge 0$. We remark that for symmetric channels (in the Gallager sense [3, p. 94]), the maximum in (12) is achieved by a uniform input distribution. We associate with the source distribution Q_S a family of tilted distributions $Q_S^{(\rho)}$ defined by

$$Q_S^{(\rho)}(s) \triangleq \frac{Q_S^{\frac{1}{1+\rho}}(s)}{\sum_{s' \in \mathcal{S}} Q_S^{\frac{1}{1+\rho}}(s')}, \qquad s \in \mathcal{S}, \qquad \rho \geq 0. \quad (14)$$

Using the results of [5], we obtain a sufficient condition for which $\underline{E}_J^{SID} > \overline{E}_J$ for binary sources and symmetric channels.

Lemma 1: Let $Q_S = \{q, 1-q\}$ ($q < 0.5$) be a binary DMS, and $W_{Y|X}$ be symmetric such that $tH_{Q_S}(S) < C(W_{Y|X})$. If $\overline{\rho}^* \leq 1$ and

$$E_o(\overline{\rho}^*, W_{Y|X}) - tE_s(\overline{\rho}^*, Q_S) < tE_o(1, Q_S^{(\overline{\rho}^*)}, Q_{L|S}) + tD(Q_S^{(\overline{\rho}^*)} \| Q_S),$$

then $E_J^{SID} > E_J$, where $\overline{\rho}^*$ achieves the maximum of $E_o(\rho, W_{Y|X}) - tE_s(\rho, Q_S)$.

Remark 1: The condition $\overline{\rho}^* \leq 1$ is to ensure that E_J is determined by the bounds $\underline{E}_J = \overline{E}_J$ [5]. The condition in Lemma 1 can be easily verified since $\overline{\rho}^*$ can be solved analytically [5, Eq. (40)].

To illustrate our results, a numerical example is next given. Let the transmitted source Q_S be a binary DMS with distribution $Q_S = \{q, 1-q\}$ ($q < 0.5$), and let the channel $W_{Y|X}$ be a binary symmetric channel (BSC) with crossover probability $\varepsilon \in (0, 0.5)$. The source Q_L is a noisy version of Q_S described by $L = S \oplus N \mod 2$ ($\mathcal{L} = \mathcal{N} = \{0, 1\}$) with noise distribution $P_N(N = 1) = 0.05$, i.e., the side information is transmitted through a dummy BSC $Q_{L|S}$ with crossover probability 0.05. Set the transmission rate $t = 0.75$. Fig. 2 shows the regions of the binary source and the BSC parameters, i.e., (ε, q) pairs, for which the source can be reliably transmitted over the channel and E_J^{SID} can be strictly larger than E_J by Lemma 1. Region **A** (including the boundary with **B**) is the region where $tH_{Q_{SL}}(S|L) \geq C(W_{Y|X})$, i.e., where both E_J^{SID} and E_J are zero. Region **B** (including the boundary with **C**) is the region where $tH_{Q_{SL}}(S|L) < C(W_{Y|X}) \leq tH_{Q_S}(S)$, i.e., where E_J is zero, but E_J^{SID} is positive. Region **C** (not including the boundary with **D**) is the region where both E_J^{SID} and E_J are positive, but the condition given in Lemma 1 holds, i.e., $E_J^{SID} > E_J > 0$. In Region **D**, both exponents E_J^{SID} and E_J are positive, and the condition in Lemma 1 is not satisfied. Note that Lemma 1 only gives a sufficient condition which can be easily verified. This condition is however not necessary for having $E_J^{SID} > E_J$; this is illustrated in Fig. 3, where we note that $E_J^{SID} > E_J$ for some $(\varepsilon, q) \in$ **D**.

We plot in Fig. 3 the lower bound E_J^{SID} given in (6), the sphere-packing upper bound $\overline{E}_J$ given in (2), and the random-coding lower bound $\underline{E}_J$ given in (1) for the above DMS(q)–BSC(ε) system with $q = 0.1$. The plots show that $\underline{E}_J^{SID}$ is strictly larger than $\overline{E}_J$ for $\varepsilon > 0.0045$, and $\underline{E}_J^{SID}$ coincides with $\underline{E}_J$ for $\varepsilon \leq 0.002$. We note that when the channel has large noise ($\varepsilon > 0.01$), the side information can substantially improve the error exponent. Furthermore, E_J is zero for $\varepsilon \geq 0.175$, but $\underline{E}_J^{SID}$ is still positive until $\varepsilon = 0.29$. Thus with the side information Q_L at the decoder, $E_J^{SID} > E_J$ holds for a large class of source-channel conditions. Finally, note that we do not yet know whether the error exponent E_J^{SIED} can be strictly larger than the exponent E_J^{SID}. To answer this question, we may need to establish an upper bound for E_J^{SID}. This may be considered in future research.

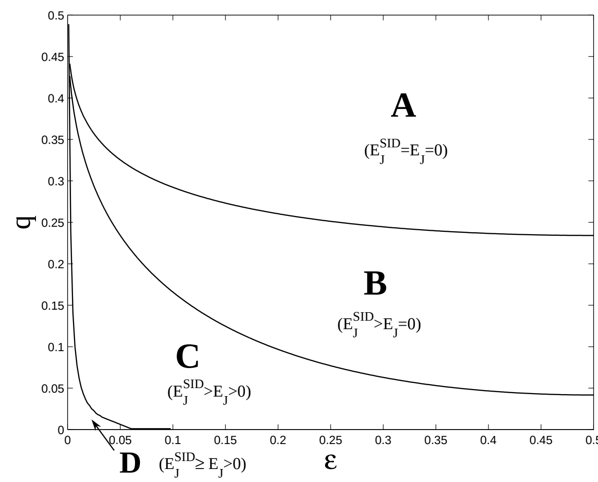

Fig. 2. Side information (SI) at the decoder can enlarge the source-channel parameters for reliable transmissibility for the binary DMS(q)–BSC(ε) system, $t = 0.75$.

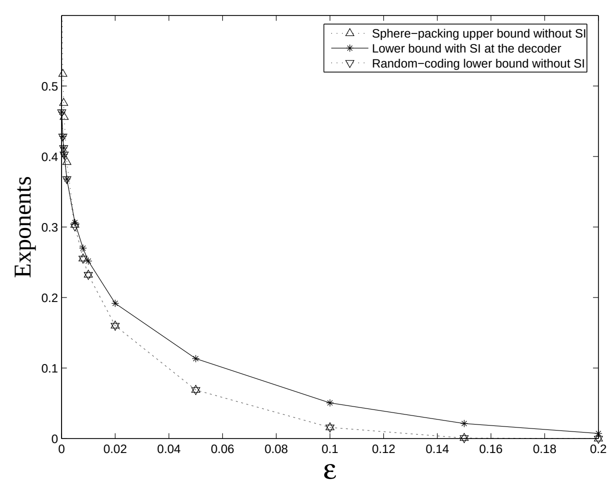

Fig. 3. Side information (SI) at the decoder can increase the JSCC error exponent for the binary DMS(q)–BSC(ε) system, $q = 0.1$, $t = 0.75$.

References

[1] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd Ed., Wiley, 2006.

[2] I. Csiszár, "Joint source-channel error exponent," *Probl. Contr. Inform. Theory*, vol. 9, pp. 315–328, 1980.

[3] R. G. Gallager, *Information Theory and Reliable Communication*, New York: Wiley, 1968.

[4] Y. Oohama and T. S. Han, "Universal coding for the Slepian-Wolf data compression system and the strong converse theorem," *IEEE Trans. Inform. Theory*, vol. 40, pp. 1908–1919, Nov. 1994.

[5] Y. Zhong, F. Alajaji, and L. L. Campbell, "On the joint source-channel coding error exponent for discrete memoryless systems," *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1450–1468, April 2006.