VQ-Based Hybrid Digital–Analog Joint Source–Channel Coding

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I. INTRODUCTION

Consider a system designed for conveying a d-dimensional random source vector, $\mathbf{X}$. A sample, $\mathbf{x}$, from the source is fed to the *encoder* ε, producing an index $i = \varepsilon(\mathbf{x}) \in \{0, \dots ; N-1\}$, where $N = 2^L$. The L bits of i are then fed to a binary symmetric channel (BSC), resulting in the output j producing a codevector $\mathbf{y}_j$ from the *decoder codebook* $\{\mathbf{y}_j\}_{j=0}^{N-1}$. We assume that the BSC corresponds to a Gaussian channel with noise variance σ^2 and with binary input in $\{\pm 1\}$.

At the transmitter, the index i also chooses a codevector $\mathbf{z}_i$ from the *encoder codebook*, $\{\mathbf{z}_i\}_{i=0}^{N-1}$, and the residual vector $\mathbf{e} = \mathbf{x} - \mathbf{z}_i$ is then formed. This vector is scaled by the constant α and transmitted over a discrete-time analog-amplitude Gaussian channel. (The scaling constant α regulates the transmission power.) The received vector $\mathbf{u} = \alpha \cdot \mathbf{e} + \mathbf{w}$, where $\mathbf{w}$ is zero-mean Gaussian with independent components of variance σ^2, is multiplied by a re-scaling constant β and then added to the codevector $\mathbf{y}_j$, resulting in an estimate of the transmitted source vector according to

$$\hat{\mathbf{x}} = \beta \mathbf{u} + \mathbf{y}_j.$$

Hence, the reproduction $\hat{\mathbf{x}}$ is *based on information transmitted both via a digital and an analog channel.* This is the key principle behind the work of this paper. Related previous work can be found in, e.g., [1, 2].

II. SYSTEM DESIGN AND PERFORMANCE

We will now present optimality criteria for the described HDA system, resulting in a design algorithm striving to minimize the distortion $D \triangleq E\|\mathbf{X} - \hat{\mathbf{X}}\|^2$ under a constraint on the transmitted power P_a per channel use in the analog channel. More precisely, the aim of the design is to find $\varepsilon(\mathbf{x})$, $\{\mathbf{z}_i\}$, $\{\mathbf{y}_j\}$ and β such that D is minimized, under the constraint that α is chosen such that $P_a = 1$ is satisfied at all times.

Optimality for a fixed encoder. Assume that $\varepsilon(\mathbf{x})$ is known and fixed, and define

$$\tilde{\mathbf{x}}(j) \triangleq E[\mathbf{X}|J=j], \; f_{kj} \triangleq \sum_{i=0}^{N-1} \Pr(I=i|J=j)\Pr(J=k|I=i)$$

and the matrices

$$\mathbf{Y} \triangleq [\mathbf{y}_0 \cdots \mathbf{y}_{N-1}], \;\; \tilde{\mathbf{X}} \triangleq [\tilde{\mathbf{x}}(0) \cdots \tilde{\mathbf{x}}(N-1)], \text{ and } (\mathbf{F})_{kj} = f_{kj}.$$

Then the optimal encoder and decoder codebooks, $\{\mathbf{z}_i\}$ and $\{\mathbf{y}_j\}$, can be *jointly* determined, by solving the equation

$$\mathbf{Y} \cdot (\mathbf{I}_N - \gamma \mathbf{F}) = (1-\gamma)\tilde{\mathbf{X}},$$

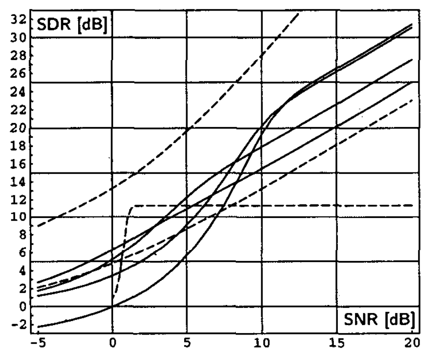
where $\mathbf{I}_N$ is the $N \times N$ unity matrix and $\gamma \triangleq \alpha\beta$, and then letting $\mathbf{z}_i = \mathbf{m}_y(i) \triangleq E[\mathbf{y}_J|I=i]$. Furthermore, the optimal β can be found (independently) as $\beta = \alpha^{-1}/(1+\sigma^2)$.

Optimality for a fixed decoder codebook. Now assume that $\{\mathbf{y}_j\}$ is given, that $\{\mathbf{z}_i\}$ is chosen as $\mathbf{z}_i = \mathbf{m}_y(i)$, and that $\beta = \alpha^{-1}/(1+\sigma^2)$, as above. The optimal encoder then is

$$\varepsilon(\mathbf{x}) = \arg\min_l \left\{(1-\gamma)\cdot\|\mathbf{x} - \mathbf{m}_y(l)\|^2 + g_l\right\},$$

where $g_i \triangleq E\left[\|\mathbf{y}_J\|^2 | I=i\right] - \|\mathbf{m}_y(i)\|^2$. Based on these results, the system can be (locally) optimized *at an assumed channel* SNR, $1/\sigma^2$, using an iterative approach similar to the well-known generalized Lloyd algorithm for VQ design.

Motivated by a broadcast scenario, we illustrate below the performance (signal-to-distortion ratio versus SNR) of employing a *fixed encoder* and an *adaptive decoder* (adapts to a varying SNR), denoted by FE$_*$AD where $*$ is the design SNR of the encoder. We also illustrate some benchmark schemes. All systems use a rate of two channel uses per source sample. The source is Gauss-Markov with correlation 0.9.



Dashed lines from above at SNR = 15 dB: The Shannon bound (distortion-rate function evaluated at channel capacity); a purely analog system (transmits each source sample twice, minimum mean-square error receiver); a purely digital tandem system (source-optimized VQ with $d = 8$ and $L = 8$, rate–1/2 Turbo code with $(n, k) = (2048, 1024)$ and generators $(37, 21)$). *Solid lines from above* at SNR = 15 dB: A HDA system with source-optimized VQ, and; HDA-FE$_*$AD systems with $* = 10, 5, 0$ dB. All HDA systems use $d = 8$ and $L = 8$.

We observe that the HDA systems outperform the tandem system and the analog system (at high SNRs). In particular we note the *graceful improvement* of the HDA systems, as opposed to the *leveling-off* in performance of the tandem system. We also observe that the performance can be improved at low SNRs using the optimization procedure.

REFERENCES

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