MAP Detection of Binary Sources Over Discrete Markov Channels†

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Extended Abstract

A source with memory as well as a memoryless source with a non-uniform distribution are sources with *redundancy*. If a discrete alphabet source is a uniformly distributed *iid* random process, it contains a maximal amount of information and exhibits no redundancy. Its entropy rate is then maximized and is equal to $\log_2 q$ bits/sample, where q is the source alphabet size. The amount of redundancy a q-ary alphabet source possesses is equal to the difference between $\log_2 q$ and its entropy rate [1].

We consider a binary source with an inherent redundancy that we make no attempt to eliminate. The source is directly transmitted over a discrete noisy channel. Our objective is therefore to design an optimum receiver which fully utilizes the source redundancy in order to combat the impairments introduced by the channel noise.

The channel considered is a binary channel with additive noise modeled according to a finite version of the Polya contagion urn scheme [2]. The errors in this channel propagate in a fashion similar to the spread of a contagious disease through a population; the occurrence of each "unfavorable" event (i.e., an error) increases the probability of future unfavorable events. The resulting noise process is a stationary mixing homogeneous Markov process with memory (order) M. The motivation for the use of such a channel is founded in the fact that real-world communication channels often have memory; our contagion-based model offers an interesting and less complex alternative to the Gilbert model and others [3].

We first investigate the problem of detecting a binary iid non-uniform source transmitted across the contagion Markov channel of order one ($M = 1$). The optimum receiver that minimizes the probability of error is a *maximum a posteriori* (MAP) detector. In a manner similar to the use of channel codes for error correction, the redundancy, due here to the non-uniform distribution of the source, is used by the MAP detector to provide some protection against channel errors. We present two MAP formulations: a *sequence* MAP detection which involves a large delay, and an *instantaneous* MAP detection which involves no delay. In sequence MAP detection, we determine the most probable transmitted *sequence* or *vector* given a vector received at the channel output. In instantaneous MAP detection, we estimate the most probable transmitted *symbol* at a particular time given all the received output symbols up to that time. The solution of the first problem results in a "Viterbi-like" implementation while the latter problem yields a recursive implementation. Sufficient as well as necessary conditions for which the sequence MAP detector is not useful are derived. These results are in the same spirit as previous results on MAP detection of Markov sources over discrete memoryless channels [4]. Simulation results for different values of the source and channel parameters, as well as for higher orders of the Markov noise process ($M = 1, 2, 5$) indicate an improvement in the performance of the MAP detectors as the channel capacity increases. We also show that for channels with relatively high bit error rates (e.g., digital cellular channels) the performance of this system (with low complexity) is superior to that of a traditional tandem source-channel coding scheme where the source and channel codes are separately designed with the assumption that the Markov channel is rendered memoryless by means of an interleaver and deinterleaver.

We then analyze the same detection problem with the variation that the source is a binary symmetric Markov source. In this case, the redundancy in the source is introduced by the Markov interdependency between successive source symbols. As for the case of the binary iid source, the two MAP detection problems can be implemented using a modified version of the Viterbi decoding algorithm and a recursive algorithm. Analytical as well as simulation results show the existence of a "mismatch" between the source and the channel. This mismatch is reduced by the use of a rate-one convolutional encoder [1]. Finally, we generalize the detection problem for the case of a binary non-symmetric Markov source.

Applications of the MAP detection problem in a combined source-channel coding system are currently under investigation. Future work may consist of comparing the results above to those obtained by detecting binary sources over the Gilbert-Elliott channel with potential applications to the digital cellular channel.

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