A Vector Quantizer for Additive White and Colored Gaussian Noise Channels[1]

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Most of the recent works (e.g., [1], [2], [3]) on joint source-channel coding deal with *discrete* channel models. In this paper, we investigate the design of joint source-channel coding scheme for *continuous-amplitude* channels. More specifically, we propose a channel-optimized vector quantizer (COVQ) [1] for additive white Gaussian noise (AWGN) and additive colored Gaussian noise (ACGN) channels with soft-decision binary phase-shift keying (BPSK) modulation.

The proposed system is as follows. The input source is a k-dimensional real vector, and the COVQ operates at a rate of r bits per source sample. For each input vector, the encoder produces kr bits for transmission. Each of these kr bits is BPSK modulated, and the output is transmitted over an additive noise channel. At the receiver, each received symbol is demodulated with q-bit soft decision via a q-bit uniform scalar quantizer with quantization step Δ [4]. Thus, for each k-dimensional source vector, qkr bits are produced at the demodulator output. These bits are then passed to the COVQ decoder.

In the case of AWGN, the concatenation of the modulator, channel, and demodulator constitutes a 2^{kr}-input, 2^{qkr}-output discrete memoryless channel (DMC) [4]. This channel is equivalent to a binary-input, 2^q-output DMC used kr times. Its channel transition probability matrix can hence be computed in terms of the quantization step Δ, the channel signal-to-noise ratio (SNR) and the complementary error function. It can be shown that this DMC is "weakly" symmetric and its capacity is therefore achieved by a uniform input distribution. For each channel SNR, we select the value of the quantization step Δ of the q-bit demodulator so that the capacity of the binary-input 2^q-output DMC is maximized.

We next design a COVQ for this (2^{kr}-input, 2^{qkr}-output) DMC using the algorithm described in [1]. The algorithm is an iterative algorithm which results in a locally optimal solution. In Table 1, we present numerical results for the quantization of a Gauss-Markov source with correlation coefficient 0.9 over the BPSK-modulated AWGN channel. The results are given in terms of the source signal-to-distortion ratio (SDR) in dB. In this table, the rate is $r = 2$ bits/sample and the dimension is $k = 2$. We used 80,000 training vectors in the COVQ design program. Note that the results for $q = 1$ correspond to hard-decision demodulation. In this case the DMC is derived from kr uses of a binary symmetric channel (BSC) with crossover probability $Q(\sqrt{SNR})$, where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\{-t^2/2\}\, dt$. Observe from Table 1 that the system performance increases as q increases (and that most of the gain is achieved at $q = 2$).

We next investigate the COVQ system for a BPSK-modulated ACGN channel (with noise correlation 0.9). Instead of using the standard interleaving technique for such a channel, we propose to utilize the statistical characteristics of the correlated channel noise by incorporating them in the design of the COVQ. This results in a COVQ scheme exploiting *both* the *channel memory* as well the channel *soft-decision information*. Unlike the AWGN case, the resulting discrete channel obtained by concatenating the modulator, ACGN channel and the soft-decision demodulator, is non-symmetric and has memory. Its capacity can not be easily obtained; we therefore resort to using the same value of Δ derived in the AWGN case. We estimate the $2^{qkr} \times 2^{kr}$ block transition matrix of this discrete channel using a long training colored noise sequence. This matrix is then incorporated in the COVQ design algorithm [3]. Numerical results for the Gauss-Markov source over the ACGN channel are displayed inside the brackets in Table 1. It can been observed that for identical channel SNR, *substantial* gains in SDR are achieved over the memoryless channel case, particularly for $q \geq 2$.

SNR	$q = 1$	$q = 2$	$q = 3$	$q = 4$
∞	13.52 [13.52]	13.52 [13.52]	13.52 [13.52]	13.52 [13.52]
8	11.20 [11.51]	11.45 [12.31]	11.53 [13.03]	11.58 [13.18]
7	10.04 [10.36]	10.70 [12.07]	10.86 [12.98]	10.91 [13.16]
6	8.92 [9.19]	9.72 [11.27]	9.94 [12.53]	9.99 [12.95]
5	7.94 [8.03]	8.75 [11.07]	9.01 [12.45]	9.08 [12.95]
4	6.97 [6.95]	7.70 [10.23]	7.98 [11.89]	8.05 [12.56]
3	6.03 [6.01]	6.86 [9.71]	7.14 [11.47]	7.21 [12.37]
2	5.15 [5.17]	5.86 [9.06]	6.12 [11.05]	6.19 [11.91]
1	4.34 [4.50]	5.06 [8.04]	5.25 [10.49]	5.31 [11.33]
0	3.62 [3.92]	4.42 [7.78]	4.61 [9.89]	4.66 [10.61]
-1	3.00 [3.40]	3.83 [7.15]	4.00 [9.26]	4.05 [10.26]
-2	2.47 [3.05]	3.29 [6.57]	3.45 [8.54]	3.50 [9.73]
-3	2.02 [2.68]	2.80 [6.00]	2.95 [7.84]	2.99 [9.04]

Table 1: COVQ performance for Gauss-Markov source over BPSK-modulated AWGN [ACGN] channel; $k = r = 2$.

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