Broadcasting Correlated Gaussian Sources with Bandwidth Expansion*

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Abstract— We study hybrid digital-analog (HDA) joint source-channel coding schemes for the transmission of a bivariate Gaussian source $S = (S_1, S_2)$ across a power-limited two-user Gaussian broadcast channel. User i $(i = 1, 2)$ observes the transmitted signal corrupted by Gaussian noise with power σ_i^2 and wants to estimate the ith component of the source, S_i. We consider HDA coding schemes with bandwidth expansion and analyze the region of (squared-error) distortion pairs that are simultaneously achievable. We first adapt an HDA scheme proposed by Reznic, Feder and Zamir in [1] for broadcasting a single common source and use it to provide an achievable distortion region for broadcasting correlated sources. We also consider a three-layered coding scheme, which we refer to by the HWZ scheme, and which consists of an analog layer and two layers each consisting of a Wyner-Ziv coder followed by a channel coder. We also examine numerical examples which indicate that the HWZ scheme performs similarly to the adapted Reznic-Feder-Zamir scheme. For comparison, we adapt the outer bound for the set of all achievable distortion pairs in broadcasting correlated Gaussian sources with matched source-channel bandwidth [2] to the bandwidth expansion case.

I. INTRODUCTION

In this work, we consider broadcasting correlated Gaussian sources with bandwidth expansion and aim to characterize the best pair of mean squared-error (MSE) distortion pairs that are simultaneously achievable at two receivers using hybrid digital analog (HDA) coding schemes. It is known that the separate (independent) design of source and channel coding due to Shannon does not in general lead to the optimal performance theoretically attainable (OPTA) in networks, see e.g. [3], [4]. On the other hand, for the point-to-point transmission of a single Gaussian source through an additive white Gaussian noise (AWGN) channel, it is well known (e.g., see [4], [5]) that if the channel and source bandwidths are equal, simple uncoded transmission achieves the OPTA. Uncoded (or analog) transmission in this case (and in the rest of this paper) means scaling the encoder input subject to the channel power constraint and transmitting without explicit channel coding. The optimality of uncoded transmission in some multi-user communication systems was recently shown in [6]–[8]. In order to exploit the advantages of both analog transmission and digital techniques, a family of HDA schemes were introduced in the literature, see e.g., [1], [9]–[19]. The case of broadcasting a single memoryless Gaussian source with bandwidth mismatch between the source and the channel using HDA schemes is considered in [1], [11]. Bross *et al*. [20] show that there exists a continuum of HDA schemes with optimal performance for the transmission of a Gaussian source over an average-power-limited Gaussian channel with matched bandwidth. Tian and Shamai generalize
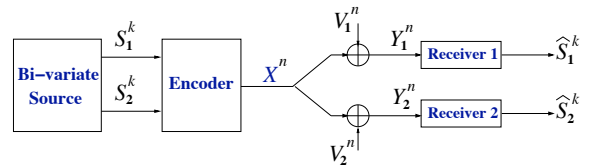
Fig. 1. Broadcasting a bivariate Gaussian source over a power-limited Gaussian two-user broadcast channel. Each receiver wants to obtain an estimate of its corresponding source component to within fidelity D_i $(i = 1, 2)$.

this result to the mismatched bandwidth case [21]. Broadcasting a Gaussian source with memory is analyzed in [16], [17]. In [22] inner and outer bounds for the distortion region in broadcasting a Gaussian mixture source are provided.

In this work, we consider the problem of broadcasting a bivariate correlated Gaussian source to two receivers subject to fidelity criteria. The system model is illustrated in Fig. 1. Related work on broadcasting correlated sources can be found in [2], [7], [23]–[29]. Lossless transmission of finite alphabet sources is considered in [23]–[27], [30], and uncoded transmission for broadcasting correlated Gaussian sources is evaluated in [7]. The problem of sending a pair of finite alphabet correlated sources through a broadcast channel with correlated side information at the receivers is studied in [27]. A lattice-based hybrid coding is propsed in [2] for broadcasting independent as well as correlated sources. The authors in [2] show that their proposed scheme is optimal for broadcasting independent sources and performs better than separate source/channel coding for broadcasting correlated sources below a certain SNR-threshold.

We aim to find achievable distortion regions using HDA schemes for broadcasting with bandwidth expansion, i.e., broadcasting with λ channel uses per source sample where $\lambda > 1$ (we specifically concentrate on $\lambda = 2$). According to our knowledge, apart from [7] and [2], in which the problem of broadcasting correlated Gaussian sources with matched bandwidth is analyzed, there are no explicit distortion-regions in the literature for broadcasting correlated Gaussian sources. We are also not aware of any previous work discussing HDA schemes for broadcasting correlated Gaussian sources with bandwidth expansion. Note that the source-channel separation theorem does not hold in this problem.

We first adapt the proposed HDA scheme for broadcasting a common source by Reznic, Feder and Zamir [1], which we refer to as the RFZ scheme, to the problem of broadcasting correlated sources and provide an achievable distortion region. Motivated by the three-layered coding scheme in [14] for broadcasting a single Gaussian source with bandwidth compression, we introduce a three-layered coding scheme for broadcasting correlated Gaussian sources with bandwidth expansion. This scheme, which we call the HWZ scheme, consists of an analog layer and two layers each consisting of a Wyner-Ziv coder

followed by a channel coder. We numerically evaluate and compare the achievable distortion regions for the two schemes. We find that, at least in the examples we consider, the HWZ scheme performs similarly to the RFZ scheme for broadcasting correlated Gaussian sources with bandwidth expansion.

The remainder of this paper is organized as follows. In Section II, we present the system model and problem statement. Section III introduces the two HDA schemes and presents the achievable distortion regions associated with them. The two regions are compared using numerical examples. Conclusions are given in Section IV.

II. PROBLEM STATEMENT

We consider broadcasting a bivariate Gaussian source (or equivalently two correlated Gaussian sources) across a two-user power-limited Gaussian broadcast channel. User 1 receives the transmitted signal corrupted by a Gaussian noise with power σ_1^2 and wants to estimate the first component of the source. User 2 observes the transmitted signal in a Gaussian noise with smaller power σ_2^2 and wants to estimate the second component of the source. Since we assume that $\sigma_1^2 > \sigma_2^2$, user 1 is referred to as the weak user and user 2 is referred to as the strong user. Let S_1 and S_2 be correlated Gaussian random variables such that $\{(S_1(t), S_2(t))\}_{t=1}^{\infty}$ is a joint stationary and memoryless Gaussian source. For each observation time $t = 1, 2, 3, ...,$ the random pair $(S_1(t), S_2(t))$ has a probability density function (pdf) $p_{S_1,S_2}(s_1, s_2) \sim \mathcal{N}(0, \Lambda)$ where the covariance matrix Λ is given by

$$\Lambda = \begin{bmatrix} \sigma_{S_1}^2 & \rho \sigma_{S_1} \sigma_{S_2} \\ \rho \sigma_{S_1} \sigma_{S_2} & \sigma_{S_2}^2 \end{bmatrix}, \quad -1 < \rho < 1. \quad (1)$$

We represent the first k instances of the first and second source components by the data sequences $S_1^k = \{S_1(1), S_1(2), \cdots, S_1(k)\}$ and $S_2^k = \{S_2(1), S_2(2), \cdots, S_2(k)\}$, respectively. The two-user Gaussian broadcast channel with receivers estimating the bivariate source components is shown in Fig. 1. Data sequences S_1^k and S_2^k are jointly encoded to $X^n = \varphi(S_1^k, S_2^k)$, where the encoder function is of the form

$$\varphi : \mathbb{R}^k \times \mathbb{R}^k \to \mathbb{R}^n. \quad (2)$$

The bandwidth expansion ratio is defined by $\lambda = \frac{n}{k}$. In [31] we have considered HDA coding schemes for broadcasting correlated Gaussian sources under both matched bandwidth ($\lambda = 1$) and bandwidth compression ($\lambda < 1$) assumptions. In this paper, we will focus on the case where $\lambda > 1$, and in particular we set $\lambda = 2$. The transmitted sequence X^n is average-power limited to $P > 0$, i.e.,

$$\frac{1}{n} \sum_{t=1}^{n} E\left[|X(t)|^2\right] \le P. \quad (3)$$

Each user i observes the transmitted signal $X(t)$ corrupted by a Gaussian noise $V_i(t)$ with power σ_i^2, so that at time t the receiver observes

$$Y_i(t) = X(t) + V_i(t), \qquad i = 1, 2 \quad (4)$$

where $V_i(t) \sim \mathcal{N}(0, \sigma_i^2)$ are independently distributed over i and t, and are independent of $X(t)$. Based on the channel output Y_i^n, receiver i provides an estimate $\widehat{S}_i^k$ of the ith component of the source, S_i^k. We consider the average MSE distortion, i.e., $\Delta_i = \frac{1}{k} E[\sum_{t=1}^{k} |S_i(t) - \widehat{S}_i(t)|^2]$. The reconstructed signal at

receiver i can be described by $\widehat{S}_i^k = \psi_i(Y_i^n)$, where decoder functions are mappings

$$\psi_i : \mathbb{R}^n \to \mathbb{R}^k, \quad i = 1, 2. \quad (5)$$

Let $\mathcal{F}^{(k,n)}(P)$ denote all encoder and decoder functions $(\varphi, \psi_1, \psi_2)$ that satisfy (2)–(5). For a particular coding scheme $(\varphi, \psi_1, \psi_2)$, the performance is determined by the channel power constraint P and incurred distortion pairs Δ_1 and Δ_2 at both receivers. For any given power constraint $P > 0$, the distortion region $\mathcal{D}$ is defined as the closure of the set of all distortion pairs (D_1, D_2) for which (P, D_1, D_2) is achievable, where a power-distortion pair (P, D_1, D_2) is achievable if for any $\delta > 0$, there exist sufficiently large integers k and $n = \lambda k$, encoding and decoding functions $(\varphi, \psi_1, \psi_2) \in \mathcal{F}^{(k,n)}(P)$, such that $\Delta_i \le D_i + \delta$ $(i = 1, 2)$.

III. DISTORTION REGIONS WITH BANDWIDTH EXPANSION

We consider the problem of broadcasting a bivariate Gaussian source with 1:2 bandwidth expansion. Joint source-channel coding schemes for broadcasting a single memoryless Gaussian source over a power-limited broadcast channel are investigated, e.g., [1], [11], [14], [16], [32], but to our knowledge no results are available for bivariate Gaussian sources other than the uncoded scheme of [7] and the lattice-based scheme of [2] which are for the matched bandwidth case. In our schemes, we will closely follow the notation and code constructions in [14]. When describing the schemes and deriving the achievable distortions, we only give high-level descriptions and analyses without detailed proofs. In particular, for brevity we treat finite-blocklength coding schemes as idealized systems with asymptotically large blocklengths. The proofs can be made rigorous by much longer arguments and by possibly assuming that the encoder and decoder have access to common randomization.

We want to transmit k samples of a bivariate Gaussian source $S^k = (S_1^k, S_2^k)$ in $n = \lambda k$ uses of a power-limited broadcast channel to two users where $\lambda = 2$. The two-user broadcast channel has the power constraint P.

A. Reznic-Feder-Zamir (RFZ) Scheme

This scheme is introduced in [1] for broadcasting a memoryless Gaussian source with bandwidth expansion. As it is mentioned in [1], Shamai *et al.*'s scheme [9] and one of the Mittal-Phamdo schemes [11] for broadcasting with bandwidth expansion are special cases of this scheme. This scheme, which provides the largest known achievable distortion region, reduces to the one proposed in [16] when specialized to the case of an independent and identically distributed (i.i.d) source and bandwidth compression. We adapt this scheme for sending correlated Gaussian sources over a Gaussian broadcast channel with bandwidth expansion. Block diagrams of the encoder and the decoder are shown in Fig. 2.

Fix P_1 and P_2 to satisfy $P = P_1 + P_2$. We first quantize the first component of the source, S_1^k, using an optimal quantizer to produce an index $m_1 \in \{1, 2, \cdots, 2^{kR_1}\}$ where $R_1 = \frac{1}{2} \log(1 + \frac{P_1}{P_2 + \sigma_1^2})$. This index is aimed to be sent to both users losslessly, i.e., it is supposed to be decoded at both users without error; thus, it is given to an optimal channel encoder. This message is also referred to as the common message [1]. Since this message must be decoded by the weak user, it is imposed that

$$\frac{k}{2}\log\left(\frac{\sigma_{S_1}^2}{D_q}\right) = \frac{n-k}{2}\log\left(1+\frac{P_1}{P_2+\sigma_1^2}\right) \qquad (6)$$

where D_q is the average distortion of the quantizer. We denote the reproduction point (i.e., output of the source decoder) by $\widehat{S}_{1d}^k$. In the analog layer, the quantization error $S_1^k - \widehat{S}_{1d}^k$ is scaled so that the power of the signal, X_a^k, in this layer is P. Specifically, at each time t, $X_a(t) = \beta(S_1(t) - \widehat{S}_{1d}(t))$ where $\beta = \sqrt{\frac{P}{D_q}}$. This codeword is transmitted as the first k component of X^n. For the remaining $n-k$ components, two coded streams are merged together. First, the quantization index m_1 is encoded treating the third layer message as a noise and the codeword X_{1d}^{n-k} with power P_1 is transmitted. The second component of the source S_2^k is then Wyner-Ziv source coded at rate $R_2 = \frac{1}{2}\log(1+\frac{P_2}{\sigma_2^2})$ using an estimate of S_1^k at the receiver as side information. The Wyner-Ziv index, $m_2 \in \{1, 2, \cdots, 2^{kR_2}\}$ is then encoded and the resulting codeword X_{2d}^{n-k} is superimposed with power P_2 such that $P_1 + P_2 = P$. The transmitted sequence is obtained by multiplexing (in time) the codeword of the analog layer X_a^k with the codeword of the digital layer, $X_d^{n-k} = X_{1d}^{n-k} + X_{2d}^{n-k}$. Thus, the transmitted sequence can be represented as $X^n = [X_a^k, X_d^{n-k}]$.

An achievable distortion-region can be obtained by varying P_1 and P_2 subject to $P = P_1 + P_2$. For a given P_1 and P_2, the achievable distortion pairs can be computed as follows. At the receiver (see Fig. 2.(b)), using the first k components of the received sequence from the analog layer, the minimum mean squared error (MMSE) estimate of the quantization error from $Y_{1a}^k = X_a^k + V_1^k$ can be obtained. Also, the common message m_1 is decoded with average distortion

$$D_q = \sigma_{S_1}^2\left(1+\frac{P_1}{P_2+\sigma_1^2}\right)^{1-\lambda}. \qquad (7)$$

Therefore, the overall average distortion at the weak user can be expressed as

$$D_1 = \frac{D_q}{1+\frac{P}{\sigma_1^2}} = \frac{\sigma_{S_1}^2}{1+\frac{P}{\sigma_1^2}}\left(1+\frac{P_1}{P_2+\sigma_1^2}\right)^{1-\lambda}. \qquad (8)$$

At the strong user, first the MMSE estimate of the quantization error from $Y_{2a}^k = X_a^k + V_2^k$ is obtained. Then, an estimate of the first component of the source can be obtained within distortion

$$D_1^* = \frac{D_q}{1+\frac{P}{\sigma_2^2}} = \frac{\sigma_{S_1}^2}{1+\frac{P}{\sigma_2^2}}\left(1+\frac{P_1}{P_2+\sigma_1^2}\right)^{1-\lambda}. $$

From this estimate, we obtain an estimate of S_2^k with distortion:

$$D_2^* = \sigma_{S_2}^2\left(1-\rho^2\left(1-\frac{D_1^*}{\sigma_{S_1}^2}\right)\right). \qquad (9)$$

This estimate acts as side information that can be used in obtaining the estimate of S_2^n for the strong user using the decoded Wyner-Ziv bits. The strong user can decode X_{1d}^{n-k} and subtract it from the received sequence to decode X_{2d}^{n-k}. Using the decoding condition for the Wyner-Ziv index m_2, the overall distortion for the strong user in estimating S_2^k can be obtained as

$$D_2 = D_2^*\left(1+\frac{P_2}{\sigma_2^2}\right)^{1-\lambda}. \qquad (10)$$

Note: If we set $\rho = 1$ and $\sigma_{S_1}^2 = \sigma_{S_2}^2$, then the results of [1], which currently appear to be the best known results for broadcasting a Gaussian source with bandwidth expansion, are obtained. Setting $P_1 = P$ reduces this scheme to one of the
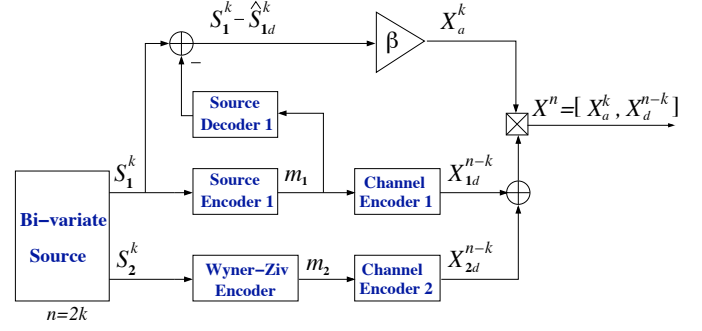
(a) Encoder
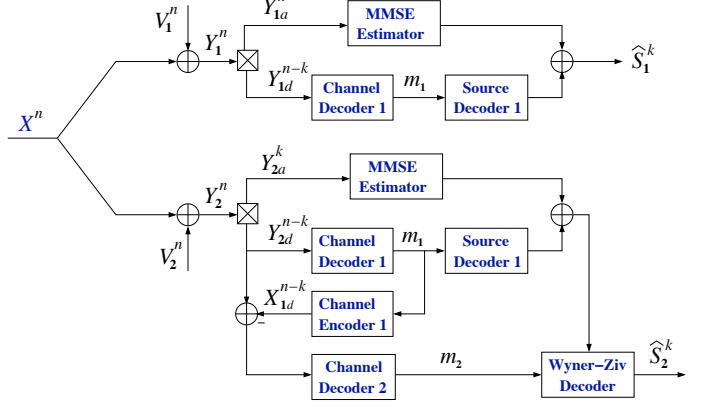
(b) Decoder

Fig. 2. Broadcasting a bivariate source $S^k = (S_1^k, S_2^k)$ with bandwidth expansion: the RFZ scheme.

Mittal-Phamdo schemes [11]. This results in

$$D_1 = \frac{\sigma_{S_1}^2}{(1+\frac{P}{\sigma_1^2})^\lambda},$$

$$D_2 = D_2^* = \sigma_{S_2}^2\left(1-\rho^2 + \rho^2\left(1+\frac{P}{\sigma_2^2}\right)^{-1}(1+\frac{P}{\sigma_1^2})^{1-\lambda}\right).$$

Here, the optimum distortion for the weak user is achieved.

If we let $P_2 = P$, the scheme of [9] is obtained, giving

$$D_1 = \frac{\sigma_{S_1}^2}{1+\frac{P}{\sigma_1^2}},$$

$$D_2 = \sigma_{S_2}^2\left(1-\rho^2\left(1-\frac{1}{1+\frac{P}{\sigma_2^2}}\right)\right)\left(1+\frac{P}{\sigma_2^2}\right)^{1-\lambda}.$$

In this scheme, if we also set $\rho = 1$, the optimum distortion for the strong user is achieved.

B. HWZ Scheme (Layering with Analog and Wyner-Ziv Coding)

This scheme comprises three layers, an analog layer and two layers each consisting of a Wyner-Ziv coder followed by a channel coder. The scheme is similar to the one proposed in [14] for broadcasting a single memoryless Gaussian source with bandwidth compression except for the following: 1) Here we consider broadcasting correlated Gaussian sources. 2) The second layer in the scheme of [14] is an HDA Costa coding while here it is a Wyner-Ziv coder followed by a channel coder. 3) Since we consider broadcasting with bandwidth expansion, only the codewords of the second layer and the third layer (digital layers) are merged together, and then the transmitted sequence is obtained by multiplexing the codeword of the analog
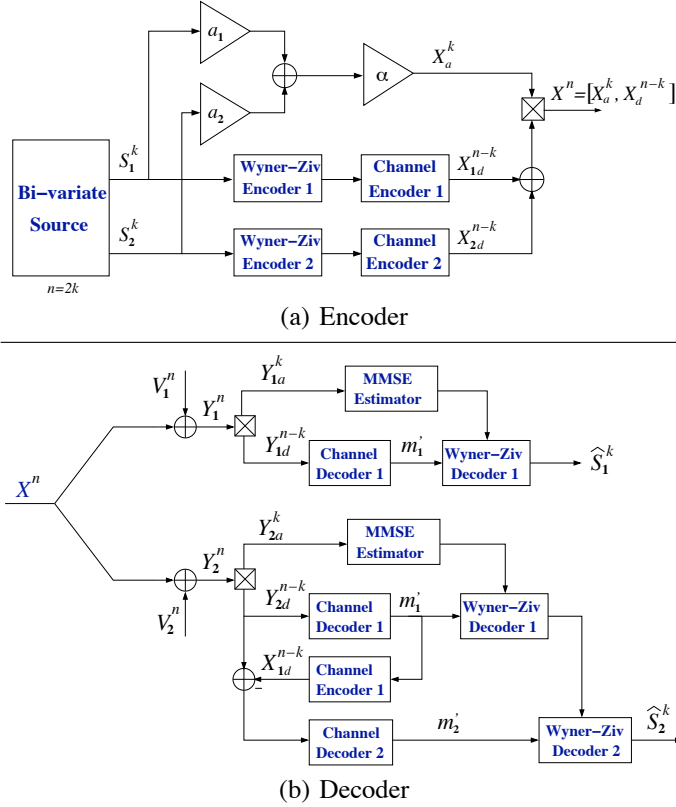
(a) Encoder

(b) Decoder

Fig. 3. Broadcasting a bivariate source $S^k = (S_1^k, S_2^k)$ with bandwidth expansion: the HWZ scheme.

layer with the codeword of the digital layer, while in [14] the codewords of all three layers are merged as bandwidth compression is examined.

Block diagrams of the encoder and the decoder are shown in Fig. 3. In the first layer, the analog transmission layer, a linear combination of the k samples of the bivariate Gaussian source components are scaled such that the power of the transmitted signal, X_a^k, in this layer is P. Thus at each time t we have $X_a(t) = \alpha \sum_{i=1}^{2} a_i S_i(t)$ where $\alpha = \sqrt{\frac{P}{a_1^2 \sigma_{S_1}^2 + a_2^2 \sigma_{S_2}^2 + 2a_1 a_2 \rho \sigma_{S_1} \sigma_{S_2}}}$. In the second layer, $n - k = k$ samples of the first component of the source, S_1^k are Wyner Ziv coded at rate $R_1' = \frac{1}{2} \log(1 + \frac{P_1}{P_2 + \sigma_1^2})$ using an estimate of S_1^k at the receiver as side information. The Wyner-Ziv index, $m_1' \in \{1, 2, \cdots, 2^{kR_1'}\}$ is then encoded treating the third layer message as a noise and the codeword X_{1d}^{n-k} with power P_1 is transmitted. In the third layer, which is meant for the strong user, the second component of the source, S_2^n, is also Wyner Ziv coded at rate $R_2' = \frac{1}{2} \log(1 + \frac{P_2}{\sigma_2^2})$ using the estimate of S_2^n at the receiver as side information. The Wyner-Ziv index, $m_2' \in \{1, 2, \cdots, 2^{kR_2'}\}$, is then encoded that treats X_{1d}^{n-k} as interference and uses power P_2 such that $P_1 + P_2 = P$. As shown in Fig. 3, the transmitted sequence is obtained by multiplexing (in time) the codeword of the analog layer X_a^k with the codeword of the digital layer, $X_d^{n-k} = X_{1d}^{n-k} + X_{2d}^{n-k}$. Thus, the transmitted sequence can be represented as $X^n = [X_a^k, X_d^{n-k}]$.

At the decoder, from the received first k components of $Y_1^n = [Y_{1a}^k, Y_{1d}^{n-k}]$, an MMSE estimate of S_1^k as $\widehat{S}_{1a}^k$ can be obtained with an average distortion

$$D_{11} = \sigma_{S_1 | \widehat{S}_{1a}}^2 = \sigma_{S_1}^2 - \frac{\alpha^2 (a_1 \sigma_{S_1}^2 + a_2 \rho \sigma_{S_1} \sigma_{S_2})^2}{P + \sigma_1^2}.$$

Since the Wyner-Ziv index m_1' must be decoded by the weak user, it is imposed that

$$\frac{k}{2} \log\left(\frac{D_{11}}{D_1}\right) = \frac{n-k}{2} \log\left(1 + \frac{P_1}{P_2 + \sigma_1^2}\right) \quad (11)$$

Therefore, the overall average distortion at the weak user can be expressed as

$$D_1 = D_{11} \left(1 + \frac{P_1}{P_2 + \sigma_1^2}\right)^{1-\lambda}. \quad (12)$$

From the analog layer, the strong user forms an estimate of the first component of the source, S_1^k with MMSE distortion

$$D_{11}^* = \sigma_{S_1}^2 - \frac{\alpha^2 (a_1 \sigma_{S_1}^2 + a_2 \rho \sigma_{S_1} \sigma_{S_2})^2}{P + \sigma_2^2}. \quad (13)$$

Then, an estimate of the first component of the source can be obtained within distortion

$$D_1^* = D_{11}^* \left(1 + \frac{P_1}{P_2 + \sigma_1^2}\right)^{1-\lambda}. \quad (14)$$

From this estimate, we obtain an estimate of S_2^k with distortion:

$$D_2^* = \sigma_{S_2}^2 \left(1 - \rho^2 \left(1 - \frac{D_1^*}{\sigma_{S_1}^2}\right)\right). \quad (15)$$

This estimate acts as side information that can be used in obtaining the estimate of S_2^n for the strong user using the decoded Wyner-Ziv bits. Using the decoding condition for the Wyner-Ziv index m_2', the overall distortion for the strong user in estimating S_2^k can be obtained as

$$D_2 = D_2^* \left(1 + \frac{P_2}{\sigma_2^2}\right)^{1-\lambda}. \quad (16)$$

In [2], [7] by assuming the knowledge of S_1^k at the receiver of the strong user, outer bound regions for broadcasting correlated Gaussian sources with matched bandwidth are provided. By making minor modifications to the proof of Theorem 1 in [2], the following result can be obtained:

Lemma 1: The distortion region for broadcasting correlated Gaussian sources with bandwidth mismatch ratio λ consists of all pairs (D_1, D_2) such that

$$\begin{aligned} D_1 &\geq \quad \sigma_{S_1}^2 \left(1 + \frac{(1-\alpha)P}{\alpha P + \sigma_1^2}\right)^{-\lambda} \\ D_2 &\geq \sigma_{S_2}^2 (1 - \rho^2) \left(1 + \frac{\alpha P}{\sigma_2^2}\right)^{-\lambda} \end{aligned} \quad (17)$$

where $\alpha \in [0, 1]$.

C. Numerical Results

We want to transmit k samples of a bivariate Gaussian source $S^k = (S_1^k, S_2^k)$ with the covariance matrix $\Lambda = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ in $n = 2k$ uses of a power-limited broadcast channel to two users (weak and strong) with observation noise variances $\sigma_1^2 = -5\,\text{dB}$ and $\sigma_2^2 = 0\,\text{dB}$, respectively. The two-user broadcast channel has the power constraint $P = 3\,\text{dB}$. The boundaries of the outer bound in Lemma 1 and of the distortion regions for both schemes are shown in Fig. 4.(a),(b) for two different values of the correlation coefficient, $\rho = 0.2$ and $\rho = 0.8$. We observe that, at least in this example, the HWZ scheme has the same performance as the adapted RFZ coding scheme.
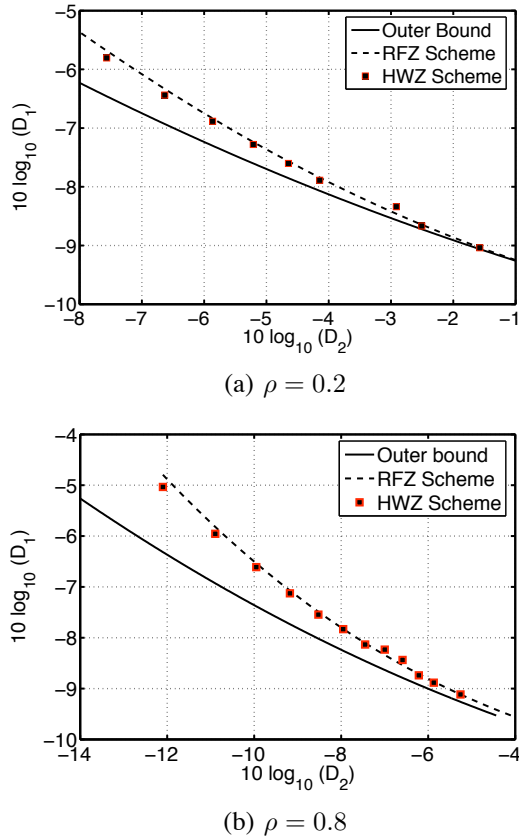
(a) $\rho = 0.2$

(b) $\rho = 0.8$

Fig. 4. Distortion regions of the different HDA coding schemes. System parameters are $\Lambda = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, $P = 3\,\mathrm{dB}$, $\sigma_1^2 = -5\,\mathrm{dB}$ and $\sigma_2^2 = 0\,\mathrm{dB}$.

IV. Conclusions

We considered HDA coding schemes for the transmission of a bivariate correlated Gaussian source over a power-limited two-user Gaussian broadcast channel. In particular, layered JSCC schemes for this problem were analyzed under bandwidth expansion assumption. We provided achievable distortion regions for two different three-layered HDA coding schemes: RFZ coding and HWZ coding. The RFZ scheme has previously been used in the literature for the broadcasting problem with a single memoryless Gaussian source. Numerical examples indicate that the scheme with analog and Wyner-Ziv coding performs similarly to the RFZ scheme in the case of bandwidth expansion. We are working on the achievable schemes to close the gap between the achievable distortion region and the outer region.

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