Source-Dependent Channel Coding of CELP Speech Over Land Mobile Radio Channels†

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ABSTRACT

We consider the problem of reliably transmitting CELP-encoded speech over a land mobile radio channel. We first quantify the "residual redundancy" inherent in the LSP's of Federal Standard 1016 CELP. This is done by modeling the quantized LSP's as first- and second-order Markov chains; these models indicate that as many as one-third of the LSP bits are redundant. We then consider methods by which that residual redundancy can be exploited by an appropriately designed channel decoder. Before transmission, the LSP's are encoded with a forward error control (FEC) code; we consider both Reed-Solomon codes and convolutional codes. Soft-decision decoders that exploit the residual correlation in the LSP's are implemented assuming a Rayleigh fading environment. Simulation results using BPSK and DQPSK modulation indicate coding gains of 2 to 5 dB over soft-decision decoders that do *not* exploit the residual correlation.

I. Introduction

We consider the problem of reliably transmitting speech encoded using the Federal Standard 1016 (FS 1016) 4.8 kbit/s code excited linear predictive (CELP) coder [3] over a land-mobile radio channel. Like all practical speech encoders, CELP does not eliminate *all* the redundancy in speech samples; what remains is "residual redundancy". In this paper, we consider methods by which channel codes can exploit that residual redundancy to enhance the quality of CELP-encoded speech over Rayleigh fading channels.

We first quantify the "residual redundancy" inherent in the line spectral parameters (LSP's) of FS 1016 CELP. Two models for the generation of LSP's are proposed; the first model incorporates only the non-uniformity of the LSP's and their correlation within a CELP frame, while the second model allows correlation between frames as well. When these models are "trained" using an actual CELP bitstream they show that as many as 12.5 of the 30 high-order LSP bits in each frame may be redundant.

We next present decoding algorithms that attempt to exploit the redundancy with both convolutional and Reed-Solomon codes. In the case of convolutional encoding, we employ three optimal soft-decision decoding schemes, all implemented using the Viterbi algorithm.

- ML – the "usual" maximum likelihood algorithm;
- MAP 1 – a MAP algorithm that exploits the redundancy due to the non-uniformity of the LSP's and their correlation *within* a frame – about 10 bits/frame;
- MAP 2 – which exploits the redundancy from the non-uniform distribution of the LSP's and their correlation within *and between* frames – about 12.5 bits/frame.

For Reed-Solomon codes, we present four soft-decision decoding (SDD) algorithms:

- SDD 1 – which approximates "traditional" maximum likelihood decoding and does not attempt to exploit any of the residual redundancy.
- SDD 2 – which exploits the redundancy due to the ordered nature of the LSP's – 4.4 bits/frame.
- SDD 3 – which like MAP 1 exploits the non-uniformity and the intra-frame correlation of the LSP's.
- SDD 4 – which like MAP 2 exploits the non-uniformity as well as the intra-frame and inter-frame correlation.

II. Channel Model

The channel is described by $y_j = a_j x_j + n_j$, where x_j, a_j, n_j and y_j are the complex channel input, Rayleigh fading, additive noise, and channel output, respectively. The additive noise, n_j, is assumed to be i.i.d. Gaussian with zero-mean and variance $N_0/2$. The fading, a_j, is correlated Rayleigh, and may be simulated by passing zero-mean i.i.d. Gaussian noise through a finite impulse response (FIR) filter [2]. The received signal is assumed to have power spectral density $S_a(f) = \frac{3}{f_g}\frac{1}{\sqrt{1-f/f_g}}$, for $f < f_g$ and $S_a(f) = 0$ for $f \geq f_g$, where f_g is the Doppler frequency.

In our simulation, we used a 201-tap FIR filter with the assumption that the normalized Doppler frequency is $T_s f_g = 0.01$, where T_s is the symbol rate.

III. LSP Residual Redundancy

In this section, we quantify the residual redundancy in the encoded LSP's of FS 1016 CELP [3]. In FS 1016 CELP, each LSP parameter is quantized by either a three-bit or a four-bit scalar quantizer. In this paper, we consider only the three most significant bits of each LSP.

Suppose we encode a segment of speech using FS 1016 CELP, resulting in a sequence of CELP frames. Let $\{U_{i,j}$:

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$1 \le i \le 10, j = 1, 2, \dots\}$ denote a random process in which $U_{i,j}$ is the i^{th} (three-bit) quantized LSP in frame j. Let $\mathbf{U}_j = [U_{1,j}, U_{2,j}, \dots, U_{10,j}]$ denote the vector consisting of the 10 quantized LSP's in frame j. If we assume that this process is stationary then the *per frame entropy rate* of this process is given by $H_\mathrm{F} = \lim_{n\to\infty} \frac{1}{n} H(\mathbf{U}_1, \mathbf{U}_2, \dots, \mathbf{U}_n)$, where $H(\mathbf{U}_1, \dots, \mathbf{U}_n)$ is the entropy of $(\mathbf{U}_1, \dots, \mathbf{U}_n)$. H_F represents the minimum number of bits per frame required to describe $\{U_{i,j} : 1 \le i \le 10, j = 1, 2, \dots\}$. If we assume $U_{i,j}$ is quantized to three bits, then the CELP encoder produces 30 bits/frame for the LSP's, meaning that the (per frame) *residual redundancy* is given by

$$\rho_T = 30 - H_\mathrm{F} \text{ (bits/frame).}$$

We seek to estimate H_F (and so ρ_T). We do this by observing a long training sequence – i.e., a realization of $\{U_{i,j} : 1 \le i \le 10, j = 1, 2, \dots\}$ – and matching the observations to a particular model of a random process; we then compute the entropy rate of the model process and use that as our estimate of H_F. The two models:

- Model A assumes that the LSP's in two different frames are independent, and the LSP's within a frame form a first-order Markov chain [1].
- Model B assumes a second-order Markov structure – that $U_{i,j}$ is independent of all the LSP's that precede it conditioned on $U_{i-1,j}$ and $U_{i,j-1}$ – the LSP that immediately precedes it in the same frame and the corresponding LSP in the frame immediately preceding [1].

Let $P_A^{(i)}(u_{i,j}|u_{i-1,j})$ and $P_B^{(i)}(u_{i,j}|u_{i,j-1}, u_{i-1,j})$ be the probability transition matrices described by models A and B, respectively.

Procedure: A large training sequence from the TIMIT speech database [4] was used; for every 30 msec of speech an LPC analysis was performed according to FS 1016 standards to arrive at the 10 quantized LSP's. The relative frequency of transitions between the values of the three high-order bits of each LSP were compiled to extract Markov transition probabilities for Model A and Model B. The entropy of the resulting Markov chains was computed to arrive at an estimate of the redundancy in each LSP and in each frame. The results are compiled in Table 1.

Model A – which does not take into account *any* correlation between frames – indicates that $\rho_T = 9.88$ of the 30 high-order bits in the LSP's are redundant. Model B – which *does* take into account both inter-frame and intra-frame correlation – indicates that $\rho_T = 12.5$ of the 30 high-order bits in the LSP's are redundant. Clearly, substantial redundancy exists within the LSP's.

IV. BPSK-Modulated Rayleigh Channels

In this section we propose soft-decision decoding algorithms over a Rayleigh fading channel used in conjunction with coherent BPSK modulation. We assume that the sequence of transmitted symbols is passed through an ideal interleaver and that we have perfect channel state information and carrier phase recovery at the receiver.

The receiver matched filter output y_j is given by

$$y_j = a_j x_j + n_j, \qquad j = 1, 2, \dots,$$

LSP Redun.	Model A			Model B		
	ρ_D	ρ_M	ρ_T	ρ_D	ρ_M	ρ_T
LSP 1	0.68	0.00	0.68	0.68	0.28	0.96
LSP 2	0.48	0.44	0.92	0.48	0.85	1.33
LSP 3	0.76	0.44	1.20	0.76	0.76	1.52
LSP 4	0.71	0.43	1.14	0.71	0.75	1.46
LSP 5	0.35	0.72	1.07	0.35	0.90	1.25
LSP 6	0.36	0.63	0.99	0.36	0.94	1.30
LSP 7	0.68	0.76	1.44	0.68	0.81	1.49
LSP 8	0.45	0.35	0.80	0.45	0.80	1.25
LSP 9	0.30	0.38	0.68	0.30	0.62	0.92
LSP 10	0.52	0.44	0.96	0.52	0.50	1.02
Frame Redun.	5.29	4.59	9.88	5.29	7.21	12.50

Table 1. Redundancy (bits/frame) of Models A and B using 83826 frames of the TIMIT speech database. ρ_D is the redundancy due to the non-uniform distribution. ρ_M is the redundancy due to memory. $\rho_T = \rho_D + \rho_M$.

where $x_j \in \{-1, +1\}$ is the BPSK-modulated transmitted bit, n_j is Gaussian with zero-mean and variance $N_0/2$, and a_j is the Rayleigh multiplicative fading.

A. Convolutional Encoding

The (three-bit) quantized LSP parameters are channel encoded by a 32-state rate-3/4 convolutional encoder with $d_\mathrm{free} = 5$ bits. Let $u_{i,j}$ denote the i^{th} LSP in frame j; let $k = 10(j-1) + i$ and re-index $u_{i,j}$ as u_k. Then the sequence of three-bits LSP's $\{u_k\}$ enter the convolutional encoder, and the encoder output is BPSK modulated; let $\mathbf{x}_k = \{+1, -1\}^4$ be the BPSK signals generated in response to u_k. Similarly, let $\mathbf{a}_k$ and $\mathbf{y}_k$ be the vectors corresponding to the fading and channel output. We consider three soft-decision decoders based on the Viterbi algorithm.

- **ML:** This algorithm is maximum likelihood decoding. The decoder chooses the code sequence $\{\mathbf{x}_k\}$ which minimizes $\sum_{k=1}^K ||\mathbf{y}_k - \mathbf{a}_k\mathbf{x}_k||^2$, where K is the number of received symbols (a multiple of 10).
- **MAP 1:** This decoder is a maximum a posteriori decoder exploiting the redundancy "captured" by Model A. The MAP 1 decoder chooses the code sequence $\{\mathbf{x}_k\}$ which minimizes
$$\sum_{k=1}^{K} ||\mathbf{y}_k - \mathbf{a}_k\mathbf{x}_k||^2 - N_0 \ln P_A^{(i)}(u_k|u_{k-1}),$$
where $\{u_k\}$ is the LSP sequence corresponding to the code sequence $\{\mathbf{x}_k\}$ and $i = k - 10\lfloor k/10 \rfloor$.
- **MAP 2:** This is similar to MAP 1 except that it exploits the redundancy described by Model B. The goal here is to minimize
$$\sum_{k=1}^{K} ||\mathbf{y}_k - \mathbf{a}_k\mathbf{x}_k||^2 - N_0 \ln P_B^{(i)}(u_k|u_{k-10}, u_{k-1}).$$

In the simulation, the decoder has a path memory of ten stages – i.e., one CELP frame. In the above algorithms, if the decoded LSP vector $\hat{U}_j$ is not ordered, we simply re-order them to yield an ordered output.

B. Reed-Solomon Encoding

We now describe four different soft-decision decoding (SDD) algorithms for block codes. In each case we assume that the ten three-bit LSP's in each frame are encoded with a $(15,10)$ code $\mathcal{C}$ over $\mathcal{F}_8$. The code we use is a concatenation of a $(9,6)$ extended Reed-Solomon code with $d_{\min}=4$ and a $(6,4)$ shortened Reed-Solomon code with $d_{\min}=3$. We then assume that the codewords are transmitted over the BPSK-modulated Rayleigh channel described above. The four soft-decision decoding algorithms make increasing use of the residual redundancy present in the LSP's. Descriptions of the decoding algorithms follow.

- **SDD 1:** This algorithm is near-maximum likelihood decoding. The decoding is in two stages. *Stage 1* generates a list of codeword "candidates" that will "compete" to be the decoder's estimate of the transmitted codeword. The matched filter outputs corresponding to the encoded LSP's are quantized into 15-tuples over $\mathcal{F}_8$ in $P=2^b$ different ways; this is done by flipping the b least confident bits. For each of the P quantizations we arrive at L different noise estimates, corresponding to the L lowest-weight vectors in the same coset as the quantized 15-tuple; the net result is (at most) $N=PL$ codeword candidates. *Stage 2* compares the distance between the received (unquantized) vector and each of the candidates on the list. If $\mathbf{y}=[y_0,y_1,\ldots,y_{44}]$ is the real 45-tuple corresponding to the (unquantized) matched filter outputs, then the codeword estimate is
$$\mathbf{c}^* = \arg\min_{\mathbf{c}\,\in\,\text{list}} \left\{ \sum_{j=0}^{44} (y_j - a_j x_j)^2 \right\}, \qquad (1)$$
where $\mathbf{x} \doteq [x_0,x_1,\ldots,x_{44}] \in \mathbf{R}^{45}$ is the BPSK-modulated codeword $\mathbf{c}=[c_0,c_1,\ldots,c_{44}]$.
- **SDD 2:** This algorithm is identical to SDD 1 with one exception: The ordering property is accounted for. During Stage 1 we generate a list of at most N codewords corresponding to *ordered* LSP's. If the list is empty, we repeat the LSP's from the previous frame.
- **SDD 3:** This algorithm is near-maximum a posteriori (near-MAP) decoding. It is identical to SDD 2 except that, during Stage 2, we do not use weighted Euclidean distance as our metric but instead use the "MAP metric" –
$$\mathbf{c}^* = \arg\min_{\mathbf{c}\,\in\,\text{list}} \left\{ \sum_{j=0}^{44} (y_j - a_j x_j)^2 - N_0 \ln(P(\mathbf{c})) \right\}. \qquad (2)$$
- **SDD 4:** This algorithm exploits *both* temporal correlation *and* correlation within a frame. Suppose we observe j (possibly corrupted) CELP frames; call them $\underline{\mathbf{Y}}=[\mathbf{Y}_1,\ldots,\mathbf{Y}_j]$, and assume we observe $\underline{\mathbf{Y}}=\underline{\mathbf{y}}=[\mathbf{y}_1,\ldots,\mathbf{y}_j]$. Then the MAP estimate of the j^{th} codeword is
$$\hat{\mathbf{X}}_j = \arg\max_{\mathbf{x}\in\mathcal{C}} f_{\underline{\mathbf{Y}}|\mathbf{X}_j,\underline{\mathbf{A}}}(\underline{\mathbf{y}}|\mathbf{x},\underline{\mathbf{a}}) P(\mathbf{X}_j=\mathbf{x}).$$
If we let the objective function to be maximized be
$$g^{(j)}(\mathbf{x}) = f_{\underline{\mathbf{Y}}|\mathbf{X}_j,\underline{\mathbf{A}}}(\underline{\mathbf{y}}|\mathbf{x},\underline{\mathbf{a}}) P(\mathbf{X}_j=\mathbf{x}),$$
then it can be maximized recursively. If we simplify the objective function by setting $f_{\underline{\mathbf{Y}}|\mathbf{X}_j,\underline{\mathbf{A}}}(\underline{\mathbf{y}}|\mathbf{x},\underline{\mathbf{a}})=0$ for codewords $\mathbf{x}$ not on the list of candidates, the result is a simple, iterative decoder that exploits *both* the inter-frame *and* the intra-frame correlation present in the LSP's.

V. DQPSK-Modulated Rayleigh Channels

We now consider the four SDD algorithms for Reed-Solomon codes over a Rayleigh fading channels without the assumption of perfect interleaving. Moreover, we consider differential QPSK as our modulation.

A CELP frame is assumed to contain 240 bits – 144 "data" bits and 96 redundant and overhead bits. We assume interleaving is done within each frame with a 6×40 interleaving table; the codeword from the $(15,10)$ Reed-Solomon code is placed in the top two rows, and bits are transmitted column by column. We select $\pi/4$-shift DQPSK as the modulation scheme in this section because it is a popular choice for land mobile radio.

In DQPSK modulation, a dibit h is conveyed by the phase difference θ_h between two consecutive baud. We assume dibits 0=00, 1=01, 2=10 and 3=11 are represented by phase changes of $135^o, -135^o, 45^o$, and -45^o, respectively. If T_s is the baud duration and dibit h is conveyed during time interval $[0,2T_s]$ by signal $s_h(t)$, then
$$s_h(t) = \begin{cases} A\cos(2\pi f_c t+\theta), & 0<t\le T_s, \\ A\cos(2\pi f_c t+\theta+\theta_h), & T_s<t\le 2T_s. \end{cases}$$
Here $h\in\{0,1,2,3\}$, and θ is an unknown phase assumed to be uniformly distributed on $[0,2\pi]$. The channel imposes both amplitude fading and phase distortion on $s_h(t)$. We denote A_1 as the faded amplitude in $[0,T_s]$ and A_2 in $[T_s,2T_s]$, and we assume the receiver can detect these faded amplitudes.

Let $s_h^*(t)$ denote the faded signal; the receiver observes the signal $r(t)$ during $[0,2T_s]$, where $r(t)=s_h^*(t)+n(t)$. The demodulator selects the dibit $\hat{h}$ such that
$$\hat{h} = \arg\max_h f_h(r(t)).$$
where $f_h(r(t))$ is the conditional pdf of the received waveform given that dibit h was transmitted. Then
$$f_h(r(t)) \;=\; \hat{F}\cdot I_0\left(\frac{2\ell_h}{N_0}\right), \qquad (3)$$
where $\hat{F}$ is independent of h, $I_0(\cdot)$ is the 0^{th} order modified Bessel function of first kind and
$$\ell_h^2 = \left[\int_0^{T_s} r(t)A_1\cos(2\pi f_c t)dt + \int_{T_s}^{2T_s} r(t)A_2\cos(2\pi f_c t+\theta_h)dt\right]^2 +$$
$$\left[\int_0^{T_s} r(t)A_1\sin(2\pi f_c t)dt + \int_{T_s}^{2T_s} r(t)A_2\sin(2\pi f_c t+\theta_h)dt\right]^2.$$
We now consider a metric for indicating the confidence with which a decision about a particular bit is made.

Consider the most significant bit (msb) of a dibit. If msb=1, then a phase change of $\pm 90^0$ is effected; if msb=0, a phase change of $\pm 135^o$ results. Based on this observation, consider the hypothetical signal

$$s_h(t) = \begin{cases} A\cos(2\pi f_c t + \theta), & 0 < t \leq T_s, \\ A\cos(2\pi f_c t + \theta + \theta_h), & T_s < t \leq 2T_s, \end{cases}$$

where $h \in \{0, 1\}$, $\theta_0 = 180^o$ and $\theta_1 = 0^o$. Following similar steps as above, we achieve the same probability expression in equation (3); i.e.,

$$f_h(r(t)) = \hat{F} \cdot I_0\left(\frac{2\ell_h}{N_0}\right). \quad (4)$$

Taking the logarithm of $f_h(r(t))$ and recognizing that $\log(I_0(x)) \approx x$ for large x, we define $\mu = (\ell_1 - \ell_0)/N_0$, and the absolute value of μ is the confidence of the most significant bit.

For the least significant bit (lsb) of a dibit, we adopt a hypothetical signal in which $\theta_0 = 90^o$ and $\theta_1 = -90^o$; the confidence for the lsb can thus be defined similarly.

Using DQPSK modulation, the four decoding algorithms for Reed-Solomon codes will be the same as described in the earlier section, except for the following changes:

- In Stage 1 of all decoding schemes, we use the μ measure to identify b the least confident bits.
- In Stage 2 of SDD 1 and SDD 2, we replace (1) by

$$\mathbf{c}^* = \arg\max_{\mathbf{c} \in \text{ list}} \left\{ \sum_{j=0}^{44} (-1)^{c_j+1} \mu_j \right\},$$

  where μ_j is the confidence of the j^{th} bit.
- In Stage 2 of SDD 3, we replace (2) by

$$\mathbf{c}^* = \arg\max_{\mathbf{c} \in \text{ list}} \left\{ \sum_{j=0}^{44} (-1)^{c_j+1} \mu_j + \ln(P(\mathbf{c})) \right\}.$$

- For SDD 4, we take $f^*_{\mathbf{Y}_j|\mathbf{X}_j,\mathbf{A}_j}(\mathbf{y}_j|\mathbf{x},\mathbf{a})$ as the product of the conditional bit probabilities given by (4).

VI. Simulation Results

Simulation results for the convolutionally encoded and Reed-Solomon encoded systems used in conjunction with coherent BPSK and DQPSK modulations over the Rayleigh channel are shown in Tables 2-7.

A large training sequence ($\sim$ 42 minutes of speech) was used from the TIMIT speech database [4] to estimate the prior LSP distributions needed for the MAP 1 and 2, and SDD 3 and 4 decoders. The testing sequence consisted of 4753 frames ($\sim$ 2.2 minutes) - 48 sentences, half uttered by female speakers and half by males from different dialect regions. No speaker appeared in both the training and testing sequences.

To evaluate the performance of the decoders we use two criteria. The first is the average spectral distortion (SD):

$$SD = \frac{1}{T}\sum_{t=1}^{T}\left[\int_{-\pi}^{\pi}\left(10\log_{10} S_t(w) - 10\log_{10}\hat{S}_t(w)\right)^2 \frac{dw}{2\pi}\right]^{\frac{1}{2}},$$

where $S_t(w)$ and $\hat{S}_t(w)$ are the original and reconstructed spectra associated with frame t, and T is the number of frames. The percentage of outliers, in parentheses in Tables 2-4, indicate the fraction of frames with distortion greater than 4 dB. (Note: The spectral distortion induced by FS 1016's scalar quantizer alone – with no channel noise – is 1.50 dB with 0.08 % of outliers > 4 dB.) The second performance measure is symbol error rate – the fraction of LSP's the decoder decodes incorrectly.

Observations Regarding Results:

- Tables 2-4 give the average spectral distortion for the four proposed algorithms at different values of E_c/N_0, where E_c is the average energy per transmitted bit. The decoding algorithms that exploit the most residual correlation – SDD 4 and MAP 2 – substantially outperform the algorithms that do not exploit any – SDD 1 and ML; this especially holds at low SNR. Note, for example, that to attain $SD = 2.75$ dB, a convolutional code with ML decoding requires $E_c/N_0 = 5$ dB, while if MAP 2 is used only $E_c/N_0 = 0$ dB is required.
- Tables 5-7 display the symbol error rates for each algorithm. Again we find that the algorithms that exploit the most residual redundancy have the lowest symbol error rate; and again this is most noticeable at low SNR.

VII. Conclusions

We conclude that line spectral parameters in FS-1016 CELP contain significant redundancy, and that this redundancy can be exploited by an appropriately designed channel decoder. Using simple first- and second-order Markov models for the quantized LSP's, we were able to design MAP and near-MAP channel decoders that provide up to 5 dB of *additional* coding gain over that obtained with maximum likelihood decoding. This gain is confirmed by listening tests presented in [1]. The tests, performed for very noisy AWGN channels, indicate a significant improvement in terms of speech quality of SDD 4 over SDD 1.

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E_c/N_0	SDD 1	SDD 2	SDD 3	SDD 4
0	5.38 (63.65%)	3.88 (38.35%)	2.97 (22.95%)	2.66 (17.86%)
1	4.41 (49.08%)	3.12 (25.75%)	2.47 (14.81%)	2.28 (11.42%)
2	3.53 (34.62%)	2.54 (16.06%)	2.12 (8.94%)	2.00 (6.59%)
3	2.86 (22.81%)	2.14 (9.30%)	1.90 (5.18%)	1.84 (4.10%)
4	2.34 (13.79%)	1.89 (5.26%)	1.76 (2.90%)	1.72 (2.21%)
5	1.99 (7.58%)	1.74 (2.59%)	1.66 (1.41%)	1.65 (1.26%)

Table 2. Reed-Solomon codes over BPSK Rayleigh channel with ideal interleaving – average spectral distortion (SD) in dB; $N = 64$, $L = 64$; values in parentheses are fraction of outliers > 4 dB.

E_c/N_0	SDD 1	SDD 2	SDD 3	SDD 4
2	6.15 (69.81%)	5.29 (56.24%)	4.29 (43.35%)	3.84 (37.87%)
3	5.44 (60.29%)	4.58 (46.15%)	3.70 (34.39%)	3.34 (28.82%)
4	4.67 (49.42%)	3.88 (36.03%)	3.23 (26.87%)	2.93 (22.47%)
5	4.02 (39.88%)	3.28 (27.81%)	2.81 (20.17%)	2.55 (16.63%)
6	3.48 (31.48%)	2.82 (20.41%)	2.47 (15.15%)	2.28 (12.21%)
7	2.99 (23.71%)	2.43 (14.56%)	2.19 (10.73%)	2.06 (8.55%)
8	2.56 (16.92%)	2.18 (10.59%)	1.99 (7.60%)	1.90 (6.08%)
9	2.29 (12.65%)	1.99 (7.56%)	1.85 (5.49%)	1.77 (4.08%)

Table 4. Reed-Solomon codes over DQPSK Rayleigh channel with finite interleaving – average spectral distortion (SD) in dB; $N = 64$, $L = 64$; values in parentheses are fraction of outliers > 4 dB.

E_c/N_0	ML	MAP 1	MAP 2
-1	82.79 %	22.76 %	17.05 %
0	79.06 %	14.28 %	9.49 %
1	71.24 %	7.52 %	4.55 %
2	58.20 %	3.59 %	2.24 %
3	40.32 %	1.46 %	0.87 %
4	22.46 %	0.71 %	0.41 %
5	9.87 %	0.28 %	0.28 %

Table 6. Convolutional codes over BPSK Rayleigh channel with ideal interleaving – symbol error rate.

E_c/N_0	ML	MAP 1	MAP 2
0	9.57 (95.24%)	3.38 (25.84%)	2.75 (17.16%)
1	8.84 (88.84%)	2.59 (14.34%)	2.18 (8.28%)
2	7.55 (76.72%)	2.09 (7.37%)	1.91 (4.26%)
3	5.79 (57.28%)	1.81 (3.17%)	1.72 (1.91%)
4	4.05 (35.36%)	1.69 (1.60%)	1.65 (0.89%)
5	2.73 (17.28%)	1.62 (0.67%)	1.62 (0.55%)

Table 3. Convolutional codes over BPSK Rayleigh channel with ideal interleaving – average spectral distortion (SD) in dB; values in parentheses are fraction of outliers > 4 dB.

E_c/N_0	SDD 1	SDD 2	SDD 3	SDD 4
-1	35.98 %	24.92 %	16.53 %	14.16 %
0	28.19 %	17.74 %	11.25 %	9.34 %
1	20.37 %	11.54 %	7.04 %	5.87 %
2	13.72 %	6.99 %	4.17 %	3.42 %
3	8.59 %	3.99 %	2.40 %	2.03 %
4	5.00 %	2.25 %	1.35 %	1.07 %
5	2.77 %	1.24 %	0.73 %	0.64 %

Table 5. Reed-Solomon codes over BPSK Rayleigh channel with ideal interleaving – symbol error rate; $N = 64$, $L = 64$.

E_c/N_0	SDD 1	SDD 2	SDD 3	SDD 4
2	36.17 %	29.38 %	23.07 %	20.82 %
3	29.69 %	23.26 %	17.76 %	15.81 %
4	23.51 %	17.78 %	13.62 %	12.12 %
5	18.21 %	13.17 %	10.07 %	8.88 %
6	14.04 %	9.54 %	7.17 %	6.32 %
7	10.36 %	6.65 %	5.04 %	4.38 %
8	7.28 %	4.74 %	3.56 %	3.07 %
9	5.27 %	3.32 %	2.39 %	2.02 %

Table 7. Reed-Solomon codes over DQPSK Rayleigh channel with finite interleaving – symbol error rate; $N = 64$, $L = 64$.