Problems 03

Due: Friday, 24 September 2021 before 17:00 EDT

1. Find all $w \in \mathbb{C}$ such that $w^2 + (-7+5\mathrm{i})w - 15\mathrm{i} = 0$. Express your solution(s) in the form $w = a + b\mathrm{i}$ where $a, b \in \mathbb{Z}$.

2. Consider the complex numbers $z := -1 - \sqrt{3}\,\mathrm{i}$ and $w := 3 - \sqrt{3}\,\mathrm{i}$.

(i) Find zw and z/w. Give your answer in the form $x + y\mathrm{i}$ where $x, y \in \mathbb{R}$.

(ii) Put z and w into polar form $re^{\theta\mathrm{i}} = r\big(\cos(\theta) + \sin(\theta)\,\mathrm{i}\big)$. Find zw and z/w using the polar form and verify that you get the same answer as in part (i).

3. Consider a triangle with side lengths a, b, and c and let α, β, and γ denote the opposite angles. Using the geometric definition of the cross product, prove the that

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}\,.$$

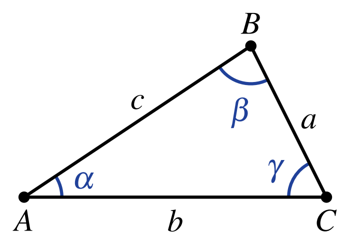

FIGURE 1. The angles α, β, γ in the triangle are opposite to the sides having length a, b, c.