Problems 14

Due: Friday, 21 January 2022 before 17:00 EST

P14.1. (i) Determine if the set $\mathbb{T} := \mathbb{R} \cup \{\infty\}$, with addition and scalar multiplication defined by

$$\boldsymbol{v} \oplus \boldsymbol{w} := \min(\boldsymbol{v}, \boldsymbol{w}) \qquad\qquad c \otimes \boldsymbol{v} := c + \boldsymbol{v}$$

for all $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{T}$ and all $c \in \mathbb{R}$, is a real vector space. If it is not, then list all of the defining axioms that fail to hold.

(ii) Determine if the set $\mathbb{P} := \{x \in \mathbb{R} \mid x > 0\}$, with addition and scalar multiplication defined by

$$\boldsymbol{v} \boxplus \boldsymbol{w} := \boldsymbol{v}\,\boldsymbol{w} \qquad\qquad c \boxtimes \boldsymbol{v} := \boldsymbol{v}^c$$

for all $\boldsymbol{v}, \boldsymbol{w} \in \mathbb{P}$ and all $c \in \mathbb{R}$, is a real vector space. If it is not, then list all of the defining axioms that fail to hold.

P14.2. Give an example of a nonempty subset U in $\mathbb{R}^2$ such that U is closed under scalar multiplication, but U is not a linear subspace of $\mathbb{R}^2$.

P14.3. Let V be a $\mathbb{K}$-vector space. Prove that the intersection of *any* set of linear subspaces in V is also a linear subspace.

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