Problem Set #5
Due: Thursday, 13 October 2011

1. Assuming that $\lim\limits_{x \to 0} \dfrac{\sin(x)}{x} = 1$ and $\lim\limits_{x \to 0} \dfrac{\ln(1+x)}{x} = 1$, compute the following limits.

(a) $\lim\limits_{\theta \to 0} \left(\dfrac{1}{\theta \tan(\theta)} - \dfrac{1}{\theta \sin(\theta)} \right)$

(b) $\lim\limits_{z \to \infty} \left(1 - \dfrac{4}{z+3} \right)^{z-2}$

2. Find all values of the parameter α for which the following function is continuous on its domain.

$$f(t) := \begin{cases} 7 - e^{t-1} & t \geq 1 \\ \alpha t^2 + t + 1 & t < 1 \end{cases}$$

3. Show that there exists a real number x such that $\sin(x) = x - 1$.