

Problem Set #6

Due: Thursday, 20 October 2011

1. If $E(v)$ is the fuel efficiency, measured in kilometres per litre ($\text{km}\cdot\text{L}^{-1}$), of a car going v kilometers per hour ($\text{km}\cdot\text{h}^{-1}$), then what are the units of $E'(100)$? What is the practical meaning of the statement $E'(80) = -2.1$?

2. Consider the function $f(x) := \frac{1}{x}$.

(a) Using the definition of the derivative, show that $f'(a) = -\frac{1}{a^2}$ for $a \neq 0$.

(b) Prove that the tangent line to the curve $y = f(x)$ at the point $(a, 1/a)$ does not intersect the curve except at the point $(a, 1/a)$.

3. Find all values of the parameters α and β for which the function G is differentiable at $t = 1$.

$$G(t) := \begin{cases} \alpha t^2 + \ln(t) & t \geq 1 \\ \beta e^{t-1} - 2t & t < 1 \end{cases}$$

Hint. You may assume that $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$.