Problem Set #8

Due: Thursday, 3 November 2011

1. Boyle's Law states that, for a fixed quantity of gas at constant temperature, the pressure P and the volume V are inversely related. Thus, for some constant k

$$PV = k\,.$$

A certain quantity of gas occupies 100 cm^3 at a pressure of 2 atmospheres. The pressure is increased, while keeping the temperature constant.

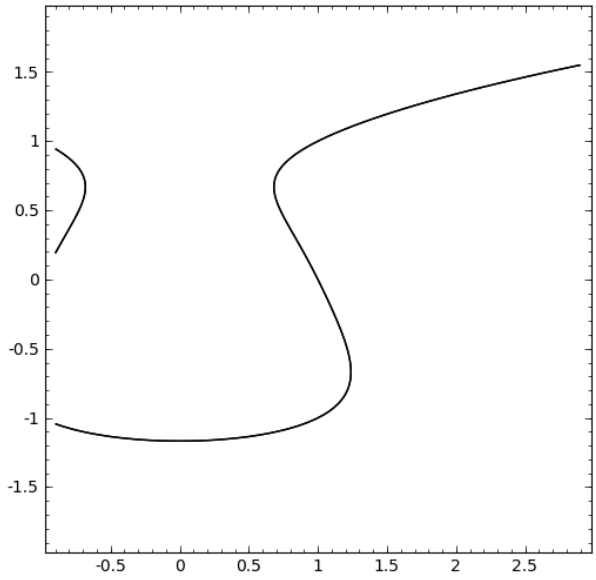
(a) Relate the rate of change of pressure and the rate of change of volume. Does the volume increase or decrease?

(b) If the pressure is increasing at a rate of 0.05 atmospheres per minute when the pressure is 2 atmospheres, find the rate at which the volume is changing at that moment. What are the units of your answer?

2. **(a)** Show that $f(x) = \dfrac{ax+b}{cx+d}$ is injective if and only if $ad - bc \neq 0$. In this case, find the inverse function of f.

(b) Suppose that g is differentiable with derivative $g'(x) = (1+x^3)^{-1/2}$. Show that the inverse function $h = g^{-1}$ satisfies $h''(x) = \frac{3}{2}[h(x)]^2$.

3. A sketch of the curve defined by the equation $y^5 - y - x^2 = -1$ appears below.



Find the equations for three different lines which are tangent to the curve when $x = 1$.