Problem Set #11

Due: Thursday, 24 November 2011

1. For the function $f(x) := \dfrac{4x^2}{x^2+1}$, do the following:

- **(a)** Find the horizontal and vertical asymptotes of $f(x)$.
- **(b)** Find f' and f''.
- **(c)** Find the critical points of f.
- **(d)** Find any inflection points.
- **(e)** Evaluate f at the critical points. Identify global maxima and minima.
- **(f)** Sketch f.

2. **(a)** For $n \in \mathbb{N}$, use induction to show that

$$\sum_{k=1}^{n} k^5 = \frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}.$$

(b) Fix $b > 0$. Use the definition of the definite integral together with the Riemann partition $P_n := \{(x_k, [x_{k-1}, x_k])\}$, where $x_k := \dfrac{kb}{n}$ for $0 \le k \le n$, to compute $\displaystyle\int_0^b x^5\,dx$.

3. Consider the definite integral $\displaystyle\int_1^2 \frac{1}{t}\,dt$.

(a) By dividing the interval $1 \le t \le 2$ into n equal parts and choosing appropriate sample points, show that

$$\sum_{j=1}^{n} \frac{1}{n+j} < \int_1^2 \frac{1}{t}\,dt < \sum_{j=0}^{n-1} \frac{1}{n+j}.$$

(b) How large should n be to approximate $\displaystyle\int_1^2 \frac{1}{t}\,dt$ with an error of at most $5 \cdot 10^{-6}$ using one of the sums in part (a)?

Hint. What is the difference between the underestimate and overestimate?