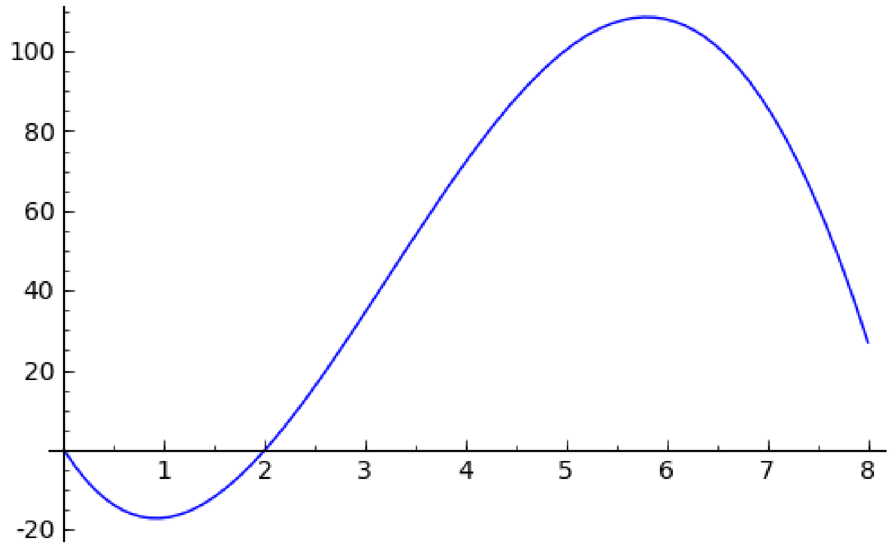
Problem Set #12

Due: Thursday, 1 December 2011

1. The graph of $f(t)$ appears below.



If $g(x) := \int_0^x f(t)\, dt$, then find the following:

(a) $g(0)$

(b) $g'(1)$

(c) The interval where g is convex.

(d) The value of x where g takes its maximum on the interval $[0,8]$.

2. **(a)** Find the derivative of the function: $H(z) = \displaystyle\int_{e^z}^{\cos(z)} \ln(w^3)\, dw$.

(b) Find all continuous functions h satisfying $\displaystyle\int_0^x h(y)\, dy = \big[h(x)\big]^2 + C$ for some constant C.

3. Let g be a differentiable function such that $g(0) = 0$ and $0 < g'(x) \leqslant 1$ for all x. For all $x \geqslant 0$, prove that

$$\int_0^x \big(g(t)\big)^3\, dt \leqslant \left(\int_0^x g(t)\, dt\right)^2.$$