Problem Set #2

Due: 19 September 2007

1. Show that the tangent lines to the path $\vec{\boldsymbol{\alpha}}\colon \mathbb{R} \to \mathbb{R}^3$ given by $\vec{\boldsymbol{\alpha}}(t) := 3t\vec{\boldsymbol{\imath}} + 3t^2\vec{\boldsymbol{\jmath}} + 2t^3\vec{\boldsymbol{k}}$ make a constant angle with the line $y = 0$, $z = x$.

2. Consider the path $\vec{\boldsymbol{\beta}}\colon (0,\pi) \to \mathbb{R}^2$ given by $\vec{\boldsymbol{\beta}}(t) := \sin(t)\vec{\boldsymbol{\imath}} + \big(\cos(t) + \ln(\tan(t/2))\big)\vec{\boldsymbol{\jmath}}$. The underlying curve is called the ***tractrix***.

(a) Show that $\vec{\boldsymbol{\beta}}$ is nonsingular at everywhere except $t = \pi/2$.

(b) Show that the length of the segment of the tangent of the tractrix between the point of tangency and the y-axis is constantly equal to 1.

Remark. The identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ may be useful in part **(a)**.

3. Let $\vec{\boldsymbol{\gamma}}\colon \mathbb{R} \to \mathbb{R}^3$ be the path defined by $\vec{\boldsymbol{\gamma}}(t) := e^{2t}\sin(t)\vec{\boldsymbol{\imath}} + e^{2t}\cos(t)\vec{\boldsymbol{\jmath}} + \vec{\boldsymbol{k}}$. The underlying curve is called a ***logarithmic spiral***.

(a) Show that $\lim\limits_{t\to-\infty} \int_t^0 \|\vec{\boldsymbol{\gamma}}\,'(u)\|\, du$ is finite; that is, the path $\vec{\boldsymbol{\gamma}}$ has finite length over the infinite interval $(-\infty, 0]$.

(b) Determine the moving frame $(\vec{\boldsymbol{T}}, \vec{\boldsymbol{N}}, \vec{\boldsymbol{B}})$ and compute the curvature and torsion for $\vec{\boldsymbol{\gamma}}$.