Problem Set #7
Due: 24 October 2008

1. **(a)** Find the volume of an ice cream cone bounded by the cone $z = \sqrt{x^2 + y^2}$ and the hemisphere $z = \sqrt{8 - x^2 - y^2}$.

   (b) Find the average distance to the origin for points in the ice cream cone region bounded by the hemisphere $z = \sqrt{8 - x^2 - y^2}$ and the cone $z = \sqrt{x^2 + y^2}$.

2. **(a)** A bead is made by drilling a cylindrical hole of radius 1 mm through a sphere of radius 5 mm. Set up a triple integral in cylindrical coordinates representing the volume of the bead. Evaluate the integral.

   (b) A *half*-melon is approximated by the region between two concentric spheres, one a radius a and the other of radius b with $0 < a < b$. Write a triple integral, including limits of integration, giving the volume of the half-melon. Evaluate the integral.

3. **(a)** Let B be the region $0 \le y \le x$ and $0 \le x \le 1$. Evaluate $\int_B (x+y)\, dA$ by making the change of variables $x = u + v$, $y = u - v$. Check your answer by evaluating the integral directly.

   (b) Use the change of variables $x = u - uv$, $y = uv$, to calculate $\int_R \frac{1}{x+y}\, dy\, dx$ where R is the region bounded by $x = 0$, $y = 0$, $x + y = 1$ and $x + y = 4$.