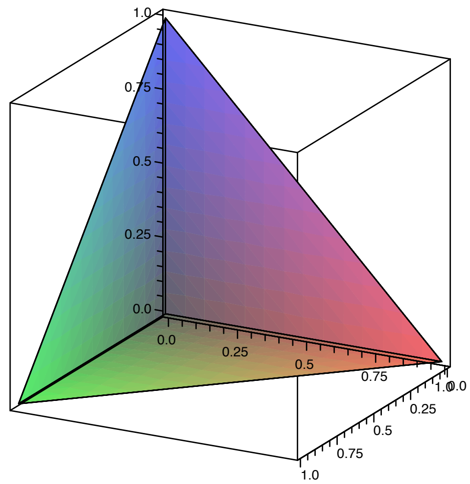
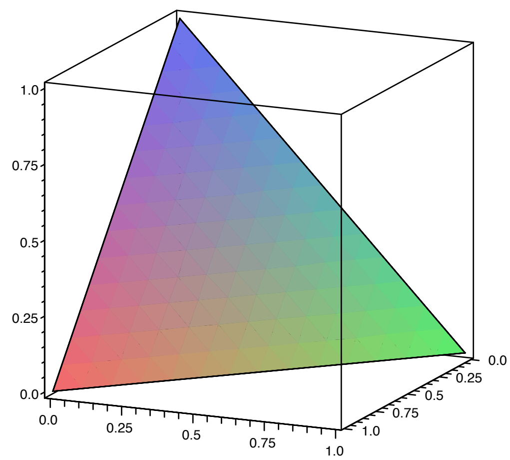
Problem Set #10

Due: 14 November 2008

1. Let P be the solid tetrahedron (a.k.a. the standard simplex in $\mathbb{R}^3$) with vertices at $\vec{\boldsymbol{p}}_0 := (0,0,0)$, $\vec{\boldsymbol{p}}_1 := (1,0,0)$, $\vec{\boldsymbol{p}}_2 := (0,1,0)$ and $\vec{\boldsymbol{p}}_3 := (0,0,1)$.





(a) Calculate the flux out of P for any constant vector field $\vec{\boldsymbol{V}} = a\vec{\boldsymbol{\imath}} + b\vec{\boldsymbol{\jmath}} + c\vec{\boldsymbol{k}}$ by computing the flux through each face separately.

(b) Explain why the answers to the first part makes sense.

2. Let $\vec{\boldsymbol{H}}(x,y,z) := (e^{xy} + 3z + 5)\vec{\boldsymbol{\imath}} + (e^{xy} + 5z + 3)\vec{\boldsymbol{\jmath}} + (3z + e^{xy})\vec{\boldsymbol{k}}$. Calculate the flux of $\vec{H}$ through the square S of side length 2 with one vertex at the origin, one edge along the positive y-axis, one edge in the xz-plane with $x > 0$, $z > 0$ and normal $\vec{\boldsymbol{n}} = \vec{\boldsymbol{\imath}} - \vec{\boldsymbol{k}}$.

3. **(a)** The torus T can be parametrized by $\vec{\boldsymbol{\tau}} \colon [0,2\pi] \times [0,2\pi] \to \mathbb{R}^3$ where $a > b > 0$ and $\vec{\boldsymbol{\tau}}(\theta,\phi) = \big(a + b\cos(\theta)\big)\cos(\phi)\vec{\boldsymbol{\imath}} + \big(a + b\cos(\theta)\big)\sin(\phi)\vec{\boldsymbol{\jmath}} + b\sin(\theta)\vec{\boldsymbol{k}}$. Find the surface area of T.

(b) Find the area of the ellipse E on the plane $2x + y + z = 2$ cut out by the circular cylinder $x^2 + y^2 = 2x$.