Problem Set #11

Due: 21 November 2008

1. Evaluate $\int_Q \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{S}}$ where $\vec{\boldsymbol{E}}(x,y,z) := ze^{x^2}\vec{\boldsymbol{\imath}} + 3y\vec{\boldsymbol{\jmath}} + (2 - yz^7)\vec{\boldsymbol{k}}$ and Q is the union of the five "upper" faces of the unit cube $[0,1] \times [0,1] \times [0,1]$ orient outward. The face $z = 0$ is *not* part of Q.

2. Let S be the surface defined by $z = e^{1-x^2-y^2}$ with $z \geqslant 1$ oriented upward and let $\vec{\boldsymbol{H}}(x,y,z) := x\vec{\boldsymbol{\imath}} + y\vec{\boldsymbol{\jmath}} + (2-2z)\vec{\boldsymbol{k}}$. Calculate $\int_S \vec{\boldsymbol{H}} \cdot d\vec{\boldsymbol{S}}$.

3. **(a)** Consider a vector field $\vec{\boldsymbol{F}} \colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{F}}(x,y,z) = x^2 + y^2 + 3$. Find an oriented surface M such that the flux integral $\int_M \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{S}}$ is negative or explain why no such surface exists.

(b) Find the flux of the vector field $\vec{\boldsymbol{G}}(x,y,z) = xy\vec{\boldsymbol{\imath}} + yz\vec{\boldsymbol{\jmath}} + zx\vec{\boldsymbol{k}}$ out of a sphere of radius 1 centered at the origin.