# Problem Set #3

**Due: 1 October 2010**

**1.** For each of the following functions, determine if there is a value for $c$ which makes the function continuous on $\mathbb{R}^2$.

**(a)** $g(x,y) = \begin{cases} \dfrac{\cos(x^2+y^2)-1}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ c & \text{if } (x,y) = (0,0) \end{cases}$

**(b)** $h(x,y) = \begin{cases} c+y & \text{if } x \leq 3 \\ 5-x & \text{if } x > 3 \end{cases}$

**2.** Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

**(a)** Compute the partial derivatives functions $f_x\colon \mathbb{R}^2 \to \mathbb{R}$ and $f_y\colon \mathbb{R}^2 \to \mathbb{R}$.
**(b)** Are the functions $f_x$ and $f_y$ continuous on $\mathbb{R}^2$?
**(c)** Is $f$ differentiable at $(0,0)$?
**(d)** Calculate the second order mixed partial derivatives $f_{xy}(0,0)$ and $f_{yx}(0,0)$.

**3.** Let $\ell\colon \mathbb{R} \to \mathbb{R}$ be a differentiable function. If $w = \ell\Big(\dfrac{x+y}{xy}\Big)$, then show that

$$x^2\frac{\partial w}{\partial x} - y^2\frac{\partial w}{\partial y} = 0\,.$$