Problem Set #1

Due: Friday, January 12, 2007

1. Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under scalar multiplication, but U is not a subspace of $\mathbb{R}^2$.

2. Let $\mathbb{K}$ be any field and let $\mathbb{K}^{\mathbb{K}}$ denote the set of all functions from $\mathbb{K}$ to $\mathbb{K}$. The set $\mathbb{K}^{\mathbb{K}}$ is a vector space over $\mathbb{K}$ with pointwise operations:

$$(f+g)(b) := f(b) + g(b) \qquad\qquad (af)(b) := a\big(f(b)\big)$$

for $f, g \in \mathbb{K}^S$ and $a, b \in \mathbb{K}$. A function $f \in \mathbb{K}^{\mathbb{K}}$ is *even* if $f(-b) = f(b)$ for all $b \in \mathbb{K}$ and *odd* if $f(-b) = -f(b)$ for all $b \in \mathbb{K}$. Prove that the set of all even functions and the set of all odd functions are subspaces of $\mathbb{K}^{\mathbb{K}}$.

3. Let V be a vector space. Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.