Problem Set #6
Due: Friday, February 16, 2007

1. Suppose $T \in \mathrm{End}(V)$ has $\dim V$ distinct eigenvalues and that $S \in \mathrm{End}(V)$ has the same eigenvectors as T (not necessarily with the same eigenvalues). Prove that $ST = TS$.

2. Let V be a complex inner product space. For $u, v \in V$ prove that $\langle u, v\rangle = 0$ if and only if $\|u\| \leq \|u + cv\|$ for all $c \in \mathbb{C}$.

Hint. Use the orthogonal decomposition and Pythagorean Theorem.

3. Prove the *polar identities*.

(a) On a real inner product space V, show that for all $u, v \in V$, we have

$$\langle u, v\rangle = \tfrac{1}{4}\big(\|u+v\|^2 - \|u-v\|^2\big)\,.$$

(b) On a complex inner product space V, show that for all $u, v \in V$, we have

$$\langle u, v\rangle = \tfrac{1}{4}\big[\|u+v\|^2 - \|u-v\|^2 + i\big(\|u+iv\|^2 - \|u-iv\|^2\big)\big]\,.$$