Problem Set #5

Due: Friday, 13 March 2020

1. Determine all of the irreducible characters for $\mathfrak{S}_4$.

2. Consider the tableau $t := \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5\\\cline{1-2}\end{array}$, and the sets $\mathcal{A} := \{4\}$ and $\mathcal{B} := \{2,5\}$.

(a) Compute the Garnir element $g_{\mathcal{A},\mathcal{B}}$.

(b) Verify directly that $g_{\mathcal{A},\mathcal{B}}\,\vec{\mathbf{e}}_t = \vec{\mathbf{0}}$.

(c) Visualize the linear relation arising from this Garnir element in terms of graphs.

(d) Express $\vec{\mathbf{e}}_{\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 5&4\\\cline{1-2}\end{array}}$ as a linear combination of standard polytabloids.

3. **(a)** For the tableau $t := \begin{array}{|c|c|c|}\hline 1&4&5\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array}$, and the sets $\mathcal{A} := \{4\}$ and $\mathcal{B} := \{5\}$, find the linear relation on polytabloids corresponding to the Garnir element $g_{\mathcal{A},\mathcal{B}}$.

(b) For the tableau $t := \begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1} 5\\\cline{1-1}\end{array}$, and the sets $\mathcal{A} := \{1,4,5\}$ and $\mathcal{B} := \{2\}$, find the linear relation on polytabloids corresponding to the Garnir element $g_{\mathcal{A},\mathcal{B}}$.

4. **(a)** Compute the matrices of the adjacent transpositions relative to the ordered basis

$$\left(\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1} 5\\\cline{1-1}\end{array}\,,\ \begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1} 5\\\cline{1-1}\end{array}\,,\ \begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1} 5\\\cline{1-1}\end{array}\,,\ \begin{array}{|c|c|c|}\hline 1&2&5\\\hline 3\\\cline{1-1} 4\\\cline{1-1}\end{array}\,,\ \begin{array}{|c|c|c|}\hline 1&3&5\\\hline 2\\\cline{1-1} 4\\\cline{1-1}\end{array}\,,\ \begin{array}{|c|c|c|}\hline 1&4&5\\\hline 2\\\cline{1-1} 3\\\cline{1-1}\end{array} \right).$$

(b) Calculate the character of $S^{(3,1^2)}$.

5. Determine all of the irreducible characters for $\mathfrak{S}_5$.