Problem Set #1

Due: Thursday, 13 September 2012

Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. (**a**) The ***Erdős collaboration graph*** has vertices corresponding to mathematicians where two mathematicians are joined by an edge whenever they co-authored a paper together (with possibly other co-authors present). Construct the Erdős collaboration graph among the following prominent graph theorists: Noga Alon, Béla Bollobás, Fan Chung, Paul Erdős, László Lovász, Paul Seymour, and Endre Szemerédi.

Hint. The American Mathematical Society's MR Collaboration Distance feature may be useful.

(**b**) The ***Hollywood graph*** has vertices corresponding to movie actors where two movie actors are joined by an edge whenever they appeared in a movie together. Construct the Hollywood graph among the following prominent actors: Kevin Bacon, Helena Bonham Carter, Colin Firth, Anne Hathaway, Julianne Moore. Gary Oldman, and Amanda Seyfried.

Hint. The Oracle of Bacon website may be useful.

2. Use MathSciNet (through a Queen's proxy), the arXiv, and MathOverflow to answer the following questions:

(**a**) Estimate the number of journal articles published with the word "tree" in their title.

(**b**) How many combinatorics preprints were added to the e-print archives in November 2010?

(**c**) Estimate the number of research level math questions tagged with `graph-theory`.

3. (**a**) According to the website Sandbox: small simple graphs, there are how many regular graphs of order at most 8 with girth 4?

(**b**) Using both Sandbox: small simple graphs and The On-Line Encyclopedia of Integer Sequences™, determine the number of connected graphs of order 18.

4. For a positive integer n, the ***n*-cube** Q_n is the graph whose vertex set is the set of all binary n-tuples where two n-tuples are adjacent if they differ in precisely one coordinate.

(**a**) Draw Q_1, Q_2, Q_3, and Q_4.

(**b**) Determine $v(Q_n)$ and $e(Q_n)$.

(**c**) Show that Q_n is bipartite for all $n \geq 1$.

5. The ***degree sequence*** of a graph is the list of vertex degrees. A ***graphic sequence*** is a list of nonnegative integers which is the degree sequence of some graph.

(**a**) If $(d_1, d_2, \ldots, d_n)$ is graphic and $d_1 \geq d_2 \geq \cdots \geq d_n$, then prove that $\sum_{i=1}^{n} d_i$ is even and

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{j=k+1}^{n} \min(d_j, k)$$

for all $1 \leq k \leq n$.

(**b**) Show that the sequences $(7, 6, 5, 4, 3, 3, 2)$ and $(6, 6, 5, 4, 3, 3, 1)$ are not graphic.