Problem Set #4

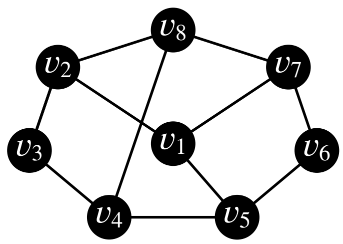
Due: Thursday, 4 October 2012

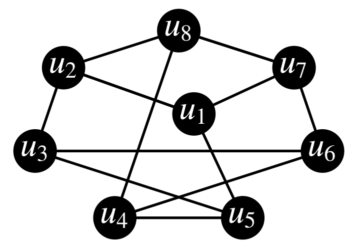
Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. Let $d_1, \ldots, d_n$ be positive integers with $n \geq 2$. Prove that there exists a tree with vertex degrees $d_1, \ldots, d_n$ if and only if $d_1 + d_2 + \cdots + d_n = 2n - 2$.

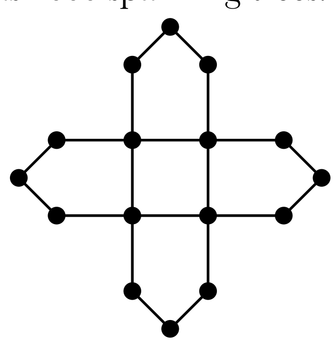
2. Suppose that T is a tree with m edges and G is a graph with $\delta(G) \geq m$. Prove that T is a subgraph of G.

3. In each graph below, find a bipartite subgraph with the maximum number of edges. Prove this is the maximum and determine whether this is the only bipartite subgraph with this many edges.





4. Show that the graph below has 2000 spanning trees.



5. Let G be the graph obtained from the complete graph K_n by deleting an edge. Use Cayley's formula to prove that the graph G has $(n-2)n^{n-3}$ spanning trees.