Problem Set #6

Due: Thursday, 18 October 2012

Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. Prove that a graph G is 2-connected if and only if G can be obtained from C_3 by a sequence of edge additions and edge subdivisions.

2. Find (with proof) a 3-regular graph with a minimal number of vertices having connectivity 1.

3. Consider the vertices $x = (0, 0, \ldots, 0)$ and $y = (1, 1, \ldots, 1)$ in the n-cube Q_n. Describe a maximum collection of internally disjoint xy-paths in Q_n and a minimal vertex-cut of Q_n separating x and y.

4. Let G be a graph such that $v(G) \geq k + 1$ and $\delta(G) \geq \frac{1}{2}(v(G) + k - 2)$. Prove that G is k-connected.

5. Let G be a graph of order n with degree sequence $d_1 \leq d_2 \leq \cdots \leq d_n$. Suppose that there exists a nonnegative integer k such that $j \leq n - 1 - d_{n-k}$ implies that $d_j \geq j + k$. Prove that G is $(k + 1)$-connected.