

## Problem Set #7

Due: Thursday, 25 October 2012

Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. Let  $G$  be a  $k$ -connected graph. If  $X$  and  $Y$  are subsets of  $V(G)$  of cardinality at least  $k$ , then show that there exists a family of  $k$  pairwise disjoint  $(X, Y)$ -paths in  $G$ .
2. If  $e$  is an edge of a cycle  $C$  in a chordal graph, then show that  $e$  forms a triangle with a third vertex of  $C$ .
3. An *interval graph* is the intersection graph of a family of intervals on the real line; it has one vertex for each interval in the family, and an edge between every pair of vertices corresponding to intervals that intersect. Prove that every interval graph is chordal.
4. Prove or disprove: Every tree has at most one perfect matching.
5. Let  $\alpha'(G)$  denote the matching number of a graph  $G$ . Prove that every maximal matching in a graph  $G$  has at least  $\alpha'(G)/2$  edges.