Problem Set #11

Due: Thursday, 22 November 2012

Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. (**a**) Calculate the chromatic polynomial of $K_{1,3}$ by using the deletion-contraction recursion $P(G,k) = P(G \setminus e, k) - P(G/e, k)$ to express it as an integer linear combinations of chromatic polynomials of empty graphs.

(**b**) Calculate the chromatic polynomial of C_4 by using the deletion-contraction recursion $P(G \setminus e, k) = P(G,k) + P(G/e,k)$ to express it as an integer linear combinations of chromatic polynomials of complete graphs.

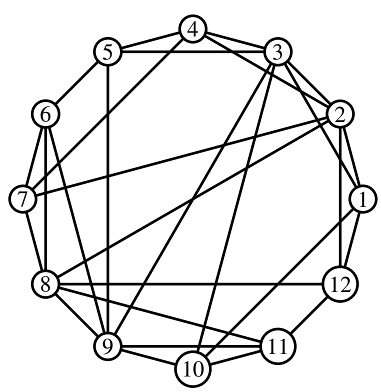
2. Let G be a connected graph with n vertices. Assume that G is not a tree. Prove that the number of k-colourings of G is less than $k(k-1)^{n-1}$ when $k \geq 3$. What happens when $k = 2$?

3. Show that every graph can be embedded in $\mathbb{R}^3$ in such a way that:

- each vertex lies on the curve $\{(t, t^2, t^3) : t \in \mathbb{R}\}$, and
- each edge is a straight line segment.

Hint. Use the Vandermonde determinant identity.

4. Find a planar embedding of the graph below in which each edge is a straight line segment.



5. A plane graph is ***self-dual*** if it is isomorphic to its dual. Show that each of the following graphs is self-dual.