Problems 08
Due: Tuesday, 10 November 2020

1. Factor $x^4 + 1$ into irreducibles in $\mathbb{F}_2[x]$, $\mathbb{F}_7[x]$, $\mathbb{F}_{13}[x]$, $\mathbb{F}_{17}[x]$, and $\mathbb{Q}[x]$.

2. (i) Determine all of the monic irreducible polynomials of degree 3 over $\mathbb{F}_3$.

(ii) Prove that
$$\frac{\mathbb{F}_3[x]}{\langle x^3 - x - 1 \rangle} \cong \frac{\mathbb{F}_3[x]}{\langle x^3 - x^2 + x + 1 \rangle} \, .$$

3. Consider $f := \det \begin{bmatrix} x & w \\ y & z \end{bmatrix} \in \mathbb{Z}[w, x, y, z]$.

(i) Prove that $\langle f \rangle$ is a prime ideal in $\mathbb{Z}[w, x, y, z]$.

(ii) Prove that $\mathbb{Z}[w, x, y, z]/\langle f \rangle$ is not a unique factorization domain.