Problem Set #2

1. Solve the initial value problem where $\frac{dy}{dx} - \frac{y}{x} = xe^x$ and $y(1) = e - 1$.

2. Find the value y_0 for which the solution of the initial value problem

$$y' - y = 1 + 3\sin(t) \qquad\qquad y(0) = y_0$$

remains finite as $t \to \infty$.

3. Solve the differential equation $(1+t^2)\frac{dx}{dt} = t^2 - 1 - 4tx$.

4. Consider a tank in which 1 g of chlorine is initially present in 100 m^3 of a solution of water and chlorine. A chlorine solution concentrated at 0.03 $\mathrm{g} \cdot \mathrm{m}^{-3}$ flows into the tank at a rate of 1 $\mathrm{m}^3 \cdot \mathrm{min}^{-1}$, while the uniformly mixed solution exits the tank at 2 $\mathrm{m}^3 \cdot \mathrm{min}$. At what time is the maximum amount of chlorine present in the tank, and how much is present?

5. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. A object at a temperature of 80°C is placed in a refrigerator maintained at 5°C. If the temperature of the object is 75°C at 20 min after it is placed in the refrigerator, determine the time (in hours) the object will reach 10°C.

6. Determine the largest interval in which the following initial value problem is certain to have a unique twice differentiable solution. Do not attempt to find the solution.

$$(x-3)y'' + xy' + \big(\ln|x|\big)y = 0 \qquad y(1) = 0 \qquad y'(1) = 1$$

7. For each pair, compute the Wronskian and determine if the functions are linearly dependents.
(a) $\{\sin^2(x) + \cos^2(x), 3\}$
(b) $\big\{x^2\cos\big(\ln(x)\big), x^2\sin\big(\ln(x)\big)\big\}$

8. Find the solution to the initial value problem where $2y'' - 3y' + y = 0$, $y(0) = 2$, and $y'(0) = \frac{1}{2}$. Determine the maximum value of the solution.

9. Solve the initial value problem

$$x'' - x' + \tfrac{1}{4}x = 0 \qquad x(0) = 2 \qquad x'(0) = A\,.$$

Determine the critical value of A that separates solutions that always remain positive from those that eventually become negative.

10. Solve the initial value problem

$$u'' + 2u' + 6u = 0 \qquad u(0) = 2 \qquad u'(0) = B \geq 0\,.$$

Find B so that $u(1) = 0$.

11. Verify that the functions 1, t, e^{-t}, and te^{-t} are solutions to $y^{(4)} + 2y''' + y'' = 0$ and determine their Wronskian.

12. Solve $y^{(4)} - 7y''' + 6y'' + 30y' - 36y = 0$.