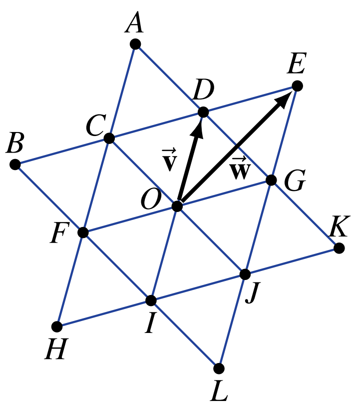
Problems 02

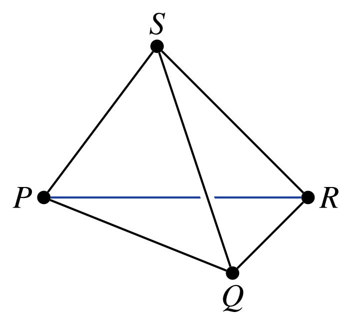
Due: Friday, 17 September 2021 before 17:00 EDT

1. The 13 points appearing in Figure 1a are part of a triangular tiling: the plane is covered by equilateral triangles with no overlaps or gaps, so each point belongs to six triangles having the same side length. Express each of the following vectors in terms of the vector $\vec{\mathbf{v}} := \overrightarrow{OD}$ and the vector $\vec{\mathbf{w}} := \overrightarrow{OE}$.

(i) $\overrightarrow{OG}$
(ii) $\overrightarrow{DG}$
(iii) $\overrightarrow{FG}$
(iv) $\overrightarrow{CJ}$
(v) $\overrightarrow{DG} + \overrightarrow{JI} + \overrightarrow{FC}$



(A) Part of a trianglar tiling



(B) Tetrahedron

2. **(i)** For any vector $\vec{\mathbf{v}}$, prove that scalar multiplication by the number 0 produces the zero vector: $0\vec{\mathbf{v}} = \vec{\mathbf{0}}$.
(ii) For any vector $\vec{\mathbf{v}}$, prove that scalar multiplication by the number -1 produces the additive inverse of $\vec{\mathbf{v}}$: $(-1)\vec{\mathbf{v}} = -\vec{\mathbf{v}}$.

3. Show that the centroid of the four vertices of a tetrahedron (a solid region bounded by four vertices, six edges joining the vertices, and four triangular faces; see Figure 1b) may also be described as follows:

(i) it is $3/4$ of the way from a vertex of the tetrahedron along the line segment joining this vertex to the centroid of the opposite face;
(ii) it is also the midpoint of the line segment joining the midpoints of any pair of opposite edges.

In particular, the centroid of a tetrahedron is the common intersection point of these seven lines.