Problems 05

Due: Friday, 8 October 2021 before 17:00 EDT

1. Find the orthogonal distance between the following skew lines in $\mathbb{R}^3$. The first line passes through the points $O \coloneqq (0,0,0)$ and $P \coloneqq (0,2,3)$, and the second line passes through the points $Q \coloneqq (4,3,3)$ and $R \coloneqq (5,5,3)$.

2. Consider the points $A \coloneqq (0,0,0)$, $B \coloneqq \big(0,0,1\big)$, $C \coloneqq \big(0,\sqrt{3}/2,1/2\big)$, and $D \coloneqq \big(\sqrt{2}/\sqrt{3},1/2\sqrt{3},1/2\big)$.

(i) Show that A, B, C, and D are all the same distance from each other.

(ii) Find the point $P \coloneqq (x,y,z)$ which is equidistant from A, B, C, and D by setting up and solving three equations in x, y, and z.

3. Solve the linear system $\left\{\begin{array}{rrrr} \mathrm{i}\,w_1 - & w_2 - & w_3 = & 1 \\ 2\mathrm{i}\,w_1 + (-2-\mathrm{i})\,w_2 + & (-2-\mathrm{i})\,w_3 = & 0 \\ -\mathrm{i}\,w_1 + & 2\,w_2 + & 2\,w_3 = & -3\mathrm{i} \end{array}\right\}$.