Problems 06
Due: Friday, 22 October 2021 before 17:00 EDT

1. Find all solutions to the linear system with augmented matrix

$$\begin{bmatrix} 2 & 3 & 1 & 3 & b_1 \\ 7 & 6 & 7 & 3 & b_2 \\ 1 & 5 & 7 & 7 & b_3 \\ 3 & 6 & 0 & 7 & b_4 \end{bmatrix}$$

where the constant term vector $\vec{b}$ is either $\begin{bmatrix} 9 \\ 23 \\ 20 \\ 16 \end{bmatrix}$ or $\begin{bmatrix} 8.92 \\ 23.01 \\ 20.01 \\ 16.03 \end{bmatrix}$.

2. For what real values of the parameters k and ℓ, does the linear system associated to the augmented matrix

$$\begin{bmatrix} 1 & -1 & -3 & 1 \\ 1 & 0 & k-3 & \ell+1 \\ 3 & k & 3k & 3 \end{bmatrix}$$

have exactly one solution, no solutions, or infinitely many solutions? When this linear system has infinitely many solutions, describe the solution set.

3. Consider a linear system $\mathbf{A}\vec{x} = \vec{b}$ where the matrix $\mathbf{A}$ and vector $\vec{b}$ have all real entries.
  (i) If $\mathbf{A}\vec{x} = \vec{b}$ has more than one solution, then prove that it has infinitely many solutions.
  (ii) If there is a solution with entries in the complex numbers, then show that there is also a solution with entries in the real numbers.