Problem Set #14

Due: Thursday, 19 January 2012

1. (a) Find $\int \sin(\theta)\cos(\theta)\, d\theta$.

(b) You probably solved part (a) by making the substitution $u = \sin(\theta)$ or $v = \cos(\theta)$ (If not, go back and do it that way). Now find $\int \sin(\theta)\cos(\theta)\, d\theta$ by making the *other* substitution.

(c) There is another way of finding this integral which involves the identities:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta), \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta).$$

Find $\int \sin(\theta)\cos(\theta)\, d\theta$ using one of these identities and the subsitution $w = 2\theta$.

(d) You should now have three different expressions for the indefinite integral $\int \sin(\theta)\cos(\theta)\, d\theta$. Are they really different? Are they all correct? Explain.

2. (a) Prove that if g is integrable on $[a,b]$, then $\displaystyle\int_a^b g(x)\, dx = \int_a^b g(a+b-x)\, dx$.

(b) Let f be an integrable odd function, i.e. $f(-x) = -f(x)$ for all $x \in \mathbb{R}$. Show algebraically that $\displaystyle\int_{-a}^{a} f(x)\, dx = 0$ for all real numbers a. Explain this fact geometrically.

3. Find $\int ze^z \cos(z)\, dz$. Check your answer by differentiation.