Problem Set #15

Due: Thursday, 26 January 2012

1. Let f'' be continuous such that $\int_0^{\pi} \big[f(x) + f''(x)\big] \sin(x)\, dx = 2$. If $f(\pi) = 1$, then find $f(0)$.

2. The following integral was used by David Bailey, Peter Borwein and Simon Plouffe [*On the rapid computation of various polylogarithmic constants*, Math. Comp. **66** (1997), 903-913] as a starting point in their determination of the ten billionth hexadecimal digit of the number π (it's 9). Evaluate the integral

$$\int_0^1 \frac{16(y-1)}{(y^2-2y+2)(y^2-2)}\, dy\,.$$

Hint. Use partial fractions of the form $\dfrac{Ay+B}{(y-1)^2+1}$ and $\dfrac{Cy+D}{y^2-2}$.

3. One of the most important functions in analysis is the gamma function,

$$\Gamma(x) := \int_0^{\infty} t^{x-1} e^{-t}\, dt \qquad \text{for all } x > 0.$$

(a) Use integration by parts to establish that $\Gamma(x+1) = x\Gamma(x)$.
(b) Find $\Gamma(1)$ and $\Gamma(2)$.
(c) For positive integers n, find a simple expression for $\Gamma(n)$.