Problem Set #18

Due: Thursday, 16 February 2012

1. The catenary $y = \frac{1}{2}(e^x + e^{-x})$ represents the shape of a hanging cable. Find the exact length of this catenary between $x = -1$ and $x = 1$.

2. Let $\vec{\boldsymbol{r}}(t) := ae^{-bt}\cos(t)\vec{\boldsymbol{\imath}} + ae^{-bt}\sin(t)\vec{\boldsymbol{\jmath}}$ where a and b are positive constants. The trace of $\vec{\boldsymbol{r}}(t)$ is called the ***logarithmic spiral***.

(a) Show that as $t \to +\infty$, $\vec{\boldsymbol{r}}(t)$ approaches the origin.

(b) Show that $\vec{\boldsymbol{r}}(t)$ has finite arc length on $[0, \infty)$.

3. The trace of $\vec{\boldsymbol{r}}(t) := \sin(t)\vec{\boldsymbol{\imath}} + \big\{\cos(t) + \ln\big[\tan\big(\frac{t}{2}\big)\big]\big\}\vec{\boldsymbol{\jmath}}$ where $t \in (0, \pi)$ is called a ***tractrix***. Show the length of the line segment of the tangent between the point of tangency on the tractrix and the y-axis is constantly equal to 1.