Problem Set #19

Due: Thursday, 1 March 2012

1. Reparametrize the curve $\vec{\gamma} \colon \mathbb{R} \to \mathbb{R}^2$ defined by $\vec{\gamma}(t) = (t^3+1, t^2-1)$ with respect to arc length measured from $(1,-1)$ in the direction of increasing t.

2. Let X be a nonempty set and consider a map $f \colon X \to Y$. Prove that the following are equivalent:
(a) f is injective;
(b) there exists $g \colon Y \to X$ such that $g \circ f = \mathbb{1}_X$ where $\mathbb{1}_X \colon X \to X$ is the identity map;
(c) for any set Z and any maps $h_1, h_2 \colon Z \to X$, the equation $f \circ h_1 = f \circ h_2$ implies that $h_1 = h_2$.

3. Suppose that X is a countable infinite set and that $g \colon \mathbb{N} \to X$ is a bijection. Let $x_0 := g(0)$. Prove that the function $f \colon X \to X \setminus \{x_0\}$ defined by $f(x) := g\big(g^{-1}(x)+1\big)$ is a bijection.