

Problems 03

Due: Friday, 27 January 2023 before 17:00 EST

P3.1. The *absolute value* function $|\cdot| : \mathbb{Z} \rightarrow \mathbb{N}$ is defined, for any integer m , by

$$|m| := \begin{cases} m & \text{if } m \geq 0, \\ -m & \text{if } m < 0. \end{cases}$$

- (i) Let n be a nonnegative integer. For any integer m , prove that $-n \leq m \leq n$ if and only if $|m| \leq n$.
- (ii) For any two integers m and n , show that

$$||n| - |m|| \leq |n + m| \leq |n| + |m| .$$

P3.2. Let k , m , and n be integers. Verify that $\gcd(km, kn) = |k| \gcd(m, n)$.

P3.3. Let k , m , and n be positive integers. When m^k divides n^k , establish that m divides n .