Problems 08

Due: Friday, 10 March 2023 before 17:00 EST

P8.1. Let $\theta\colon Q \to R$, $\varphi\colon R \to S$, and $\psi\colon S \to T$ be ring homomorphisms. When the compositions $\varphi\,\theta$ and $\psi\,\varphi$ are ring isomorphisms, prove that θ, φ, ψ, and $\psi\,\varphi\,\theta$ are also ring isomorphisms.

P8.2. Each quotient ring R/I in the left column of Table 1 is isomorphic to a ring S in the right column. Match each quotient ring with its isomorphic partner and prove that they are isomorphic be describing a surjective ring homomorphism $\varphi\colon R \to S$ with kernel I. The matching is neither injective nor surjective.

Table 1. Table of quotient rings and rings

R/I	S
$\dfrac{\mathbb{Z}[x]}{\langle 8, 12, x\rangle}$	$\dfrac{\mathbb{Z}}{\langle 3\rangle}$
$\dfrac{\mathbb{Q}[x]}{\langle x^2 - 2\rangle}$	$\dfrac{\mathbb{Z}}{\langle 4\rangle}$
$\dfrac{\mathbb{R}[x]}{\left\langle x - \sqrt{2}\right\rangle}$	$\dfrac{\mathbb{Z}}{\langle 8\rangle}$
$\dfrac{\mathbb{R}[x]}{\langle x^2 + x + 2\rangle}$	$\mathbb{Z}$
$\dfrac{\mathbb{R}[x]}{\langle x^2\rangle}$	$\mathbb{Q}$
$\dfrac{\mathbb{R}[x,y]}{\langle y - 1\rangle}$	$\mathbb{R}$
$\dfrac{\mathbb{R}[x,y]}{\langle y - 1, x + 9\rangle}$	$\mathbb{C}$
$\dfrac{\mathbb{R}[x,y]}{\langle y - 1, x^2 + 9\rangle}$	$\{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$
	$\mathbb{Z}[x]$
	$\mathbb{Q}[x]$
	$\mathbb{R}[x]$
	$\mathbb{C}[x]$
	$\left\{\begin{bmatrix} a & b \\ 0 & a \end{bmatrix} \,\middle\vert\, a, b \in \mathbb{R}\right\}$