Problem Set #2

Due: 24 September 2010

1. **(a)** Change each of the following points from Cartesian coordinates to cylindrical coordinates and spherical coordinates:

$$(2, 1, -2)\,, \quad (\sqrt{2}, 1, 1)\,, \quad (-2\sqrt{3}, -2, 3)\,.$$

(b) Convert the equation $\rho \sin(\phi) = 1$ from spherical coordinates to Cartesian coordinates.

(c) Let $\vec{\boldsymbol{\imath}}$, $\vec{\boldsymbol{\jmath}}$ and $\vec{\boldsymbol{k}}$ denote the standard basis in $\mathbb{R}^3$. Verify that the basis vectors for spherical coordinates, namely

$$\vec{\boldsymbol{e}}_\rho := \frac{x\vec{\boldsymbol{\imath}} + y\vec{\boldsymbol{\jmath}} + z\vec{\boldsymbol{k}}}{\sqrt{x^2+y^2+z^2}} = \sin(\phi)\cos(\theta)\vec{\boldsymbol{\imath}} + \sin(\phi)\sin(\theta)\vec{\boldsymbol{\jmath}} + \cos(\phi)\vec{\boldsymbol{k}}$$

$$\vec{\boldsymbol{e}}_\phi := \frac{xz\vec{\boldsymbol{\imath}} + yz\vec{\boldsymbol{\jmath}} - (x^2+y^2)\vec{\boldsymbol{k}}}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}} = \cos(\phi)\cos(\theta)\vec{\boldsymbol{\imath}} + \cos(\phi)\sin(\theta)\vec{\boldsymbol{\jmath}} - \sin(\phi)\vec{\boldsymbol{k}}$$

$$\vec{\boldsymbol{e}}_\theta := \frac{-y\vec{\boldsymbol{\imath}} + x\vec{\boldsymbol{\jmath}}}{\sqrt{x^2+y^2}} = -\sin(\theta)\vec{\boldsymbol{\imath}} + \cos(\theta)\vec{\boldsymbol{\jmath}},$$

are mutually orthogonal unit vectors.

2. **(a)** Consider the surface in $\mathbb{R}^3$ determined by the equation $x^2 + xy - xz = 2$. Find a function $F(x, y, z)$ such that this surface is a level set of F and find a function $f(x, y)$ such that this surface in the graph of f.

(b) Describe the surface $x^2 + y^2 = \big(2 + \sin(z)\big)^2$.

3. Using the ε-δ definition, prove that $\displaystyle\lim_{(x,y,z)\to(2,0,-1)} 3x + y\sin(z) = 6$.