Problem Set #3

Due: Friday, January 26, 2007

1. The *conjugate transpose* of a complex $(m \times n)$-matrix Z is the $(n \times m)$-matrix Z^* obtained by interchanging the rows and columns and then taking the complex conjugate of each entry. A complex $(n \times n)$-matrix Z is *Hermitian* if $Z = Z^*$.

(a) Show that the Hermitian matrices form a real vector space.

(b) Find a basis for this space and determine its dimension.

2. **(a)** Prove or disprove: there exists a basis (p_0, p_1, p_2, p_3) of $\mathbb{K}[t]_{\leq 3}$ such that none of the polynomials p_0, p_1, p_2, p_3 has degree 2.

(b) Let m be a positive integer and suppose $q_0, q_1, \ldots, q_m$ are polynomials in $\mathbb{K}[t]_{\leq m}$ such that $q_j(-5) = 0$ for $0 \leq j \leq m$. Show that $(q_0, q_1, \ldots, q_m)$ is linear dependent.

3. Suppose $T \in \operatorname{Hom}(V, \mathbb{K})$ and the vector $u \in V$ does not lie in $\operatorname{Ker}(T)$. Prove that $V = \operatorname{Ker}(T) \oplus \operatorname{Span}(u)$.