Problem Set #5
Due: Friday, February 9, 2007

1. Suppose that $a_0, \ldots, a_m$ are distinct elements in $\mathbb{K}$ and that $b_0, \ldots, b_m$ are elements in $\mathbb{K}$. Prove that there exists a unique polynomial $p \in \mathbb{K}[t]_{\leq m}$ such that $p(a_j) = b_j$ for $0 \leq j \leq m$.

2. Consider $T \in \operatorname{Hom}(\mathbb{R}[x]_{\leq 2}, \mathbb{R}[x]_{\leq 2})$ defined by

$$(Tf)(x) = \int_{-1}^{1} (x-y)^2 f(y)\, dy - 2f(0)x^2 \qquad \text{for all } f \in \mathbb{R}[x]_{\leq 2}.$$

Find all eigenvalues and eigenvectors for T.

3. Suppose n is a positive integer and $T \in \operatorname{End}(\mathbb{K}^n)$ is defined by

$$T(x_1, \ldots, x_n) = (x_1 + \cdots + x_n, \ldots, x_1 + \cdots + x_n)\ ;$$

in other words, T is the operator whose matrix (with respect to the standard basis) consists of all 1's. Find all eigenvalues and eigenvectors of T.