Problem Set #8

Due: Friday, March 9, 2007

1. Let $f(x) = \frac{1}{2}\big(x + |x|\big) = \left\{ \begin{smallmatrix} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{smallmatrix} \right.$. Find the projection of f onto $U := \mathbb{R}[x]_{\leq 2}$ using the following:

(a) Legendre polynomials and $\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)\; dx$;

(b) Chebyshev polynomials and $\langle f, g \rangle = \int_{-1}^{1} \frac{f(x)g(x)}{\sqrt{1-x^2}}\; dx$;

(c) "Normal equations" and $\langle f, g \rangle = \sum_{k=-3}^{3} f\big(\frac{k}{3}\big) g\big(\frac{k}{3}\big)$.

2. Find $g \in \mathbb{R}[t]_{\leq 2}$ such that $\int_0^1 f(t)\cos(\pi t)\; dt = \int_0^1 f(t)g(t)\; dt$ for all $f \in \mathbb{R}[t]_{\leq 2}$.

3. Let V be the $\mathbb{R}$-subspace of $C^2([0,1])$ consisting of all f satisfying $f(0) = f(1) = 0$. Suppose $S\colon V \to C([0,1])$ is defined by $S(f) = f'' + f'$.

(a) If $\langle f, g \rangle = \int_0^1 f(t)g(t)\; dt$, then find S^*.

(b) If $\langle f, g \rangle = \int_0^1 f(t)g(t)e^t\; dt$, then find S^*.