Problem Set #10

Due: Thursday, 15 November 2012

Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. For positive integers m and n, the ***Kneser graph*** $KG_{n,m}$ has one vertex for each m-subset of $[n] := \{1, \dots, n\}$ and two vertices are adjacent if the corresponding subsets are disjoint. Show that $\chi(KG_{2n+k,n}) \leq k + 2$.

2. The ***complement*** of a graph G is the graph $\overline{G}$ on the same vertices such that two vertices of $\overline{G}$ are adjacent if and only if they are not adjacent in G. Prove the following:

(**a**) $\chi(G) \cdot \chi(\overline{G}) \geq v(G)$;
(**b**) $\chi(G) + \chi(\overline{G}) \geq 2\big(v(G)\big)^{1/2}$;
(**c**) $\chi(G) + \chi(\overline{G}) \leq v(G) + 1$.

3. Let G be a bipartite graph. Prove that $\chi(\overline{G}) = \omega(\overline{G})$.

4. Let G be a 4-critical graph having a vertex cut S of cardinality 4. Prove that the induced subgraph $G[S]$ has at most four edges.

5. Find a smallest imperfect graph G such that $\chi(G) = \omega(G)$.