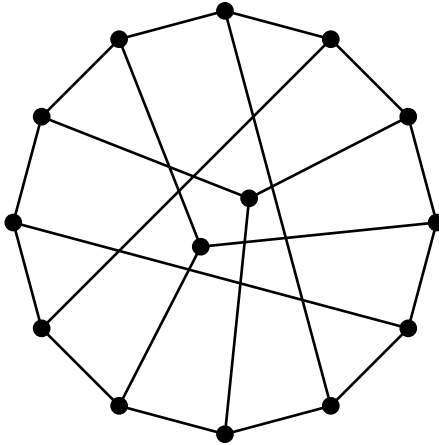


Problem Set #12

Due: Thursday, 29 November 2012

Students registered in MATH 401 should submit solutions to three of the following problems. Students in MATH 801 should submit solutions to all five.

1. For the graph below, prove it is nonplanar or provide a planar embedding.



2. (a) Show that the complement of a planar graph with at least eleven vertices is nonplanar.
(b) Find a plane graph with eight vertices whose complement is planar.
3. The *girth* of a graph is the length of the shortest cycle it contains. Let G be a connected planar graph with girth k where $k \geq 3$. Show that

$$e(G) \leq \frac{k(v(G) - 2)}{k - 2}.$$

Use this to prove that the Petersen graph is nonplanar.

4. Prove that every 3-connected graph with at least 6 vertices that contains a subdivision of K_5 also contains a subdivision of $K_{3,3}$.
5. A graph is *outerplanar* if it has a planar embedding with every vertex on the boundary of the unbounded face. Without using the Four Colour Theorem, prove that every outerplanar graph is 3-colourable.