# Problem Set #3

**1.** Solve the initial value problem where $y''' - y' = 0$, $y(0) = 0$, $y'(0) = 1$, and $y''(0) = 2$.

**2.** Solve $12u^{(4)} + 31u^{(3)} + 75u'' + 37u' + 5u = 0$.

**Hint.** Both $-\frac{1}{3}$ and $-\frac{1}{4}$ are roots of the characteristic polynomial.

**3.** Solve $x^{(8)} + 8x^{(4)} + 16x = 0$.

**Hint.** $r^4 + 4 = (r^2 - 2r + 2)(r^2 + 2r + 2)$.

**4.** A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downward velocity of $10\,\mathrm{cm} \cdot \mathrm{s}^{-1}$ and if there is no damping, determine the position of the mass at any time. When does the mass first return to its equilibrium position?

**5.** A mass of 20 g stretches a spring 5 cm. Suppose that the mass is also attached to a viscous damper with a damping constant of $400\,\mathrm{g} \cdot \mathrm{s}^{-1}$. If the mass is pulled down an additional 2 cm and then released, find its position at any time. Determine the ratio of the quasi-period to the period of the corresponding undamped motion.

**6.** Assume that the system described by the equation $my'' + cy' + ky = 0$ is either critically damped or overdamped. Show that the mass can pass through the equilibrium position at most once, regardless of the initial conditions.

**7.** Solve $u'' + u' + 4u = 2\sinh(x)$.

**Hint.** $\sinh(x) = \frac{1}{2}\left(e^x - e^{-x}\right)$.

**8.** Solve $y'' + \omega_0^2 y = \cos(\omega t)$ where $\omega^2 \neq \omega_0^2$.

**9.** Solve the initial value problem where $y'' - 2y' - 3y = 3xe^{2x}$, $y(0) = 1$, and $y'(0) = 0$.

**10.** Use the method of variation of parameters to find a particular solution to $y'' - y' - 2y = 2e^{-t}$. Check your answer by using the method of undetermined coefficients.

**11.** Solve $x'' - 2x' + x = \dfrac{e^t}{1+t^2}$.

**12.** The ***Bessel equation*** of order one-half $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$ has two linearly independent solutions $y_1(x) = x^{-1/2}\cos(x)$ and $y_2(x) = x^{-1/2}\sin(x)$. Find a general solution to the nonhomogenous equation $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = x^{5/2}$ where $x > 0$.