Department Colloquium

Speaker: Velimir Jurdjevic, University of Toronto
Date: Friday, April 1
Time: 2:30 p.m.
Place: Jeffery 234
Title: Jacobi's Geodesic Problem and Integrable Hamiltonian Systems on Lie Algebras

ABSTRACT. This lecture will introduce an affine-quadratic optimal control problem on a Lie group G with a semi-simple Lie algebra $\mathfrak{g}$ that admits Cartan decomposition $\mathfrak{g} = \mathfrak{p} + \mathfrak{k}$ subject to the usual Lie algebraic conditions

$$[\mathfrak{p}, \mathfrak{p}] \subseteq \mathfrak{k}\ ,\ [\mathfrak{p}, \mathfrak{k}] \subseteq \mathfrak{p},\ [\mathfrak{k}, \mathfrak{k}] \subseteq \mathfrak{k}.$$

We will find the necessary and sufficient conditions that the associated Hamiltonian

$$H = \frac{1}{2}\langle Q^{-1}(L_\mathfrak{k}), L_\mathfrak{k}\rangle + \langle A, L_\mathfrak{p}\rangle,$$

where $\langle\ ,\ \rangle$ denotes the Killing form, $Q : \mathfrak{k} \to \mathfrak{k}$ is a positive definite operator, and A is a regular element in $\mathfrak{p}$ admits an isospectral representation of the form

$$\frac{dL_\lambda}{dt} = [M_\lambda, L_\lambda],$$

$$M_\lambda = Q^{-1}(L_\mathfrak{k}) - \lambda A,\ L_\lambda = -L_\mathfrak{p} + \lambda L_\mathfrak{k} + (\lambda^2 - 1)B,$$

for some element $B \in \mathfrak{p}$. Then we will correlate these findings with the seminal works of S.M. Manakov, A.T. Fomenko, A.S. Mischenko, V.V. Trofimov and O. Bogoyavlensky on the integrability of the n- dimensional mechanical tops.

Additionally, we will single out an affine-quadratic Hamitonian whose spectral invariants lead to the integrals of motion for the Jacobi's geodesic problem on the ellipsoid. More explicitly, we will link Jacobi's problem with the elliptic geodesic problem on the sphere and we will be able to show that the elliptic problem on the sphere and C. Newmann's mechanical problem on the sphere share the same integrals of motion inherited from the affine Hamiltonian on the Lie algebra of matrices of zero trace.