Queen's University at Kingston
Department of Mathematics and Statistics

SUMMER 2018 Research Projects

Last updated: 1 February 2018

Supervisor: Troy Day

Project Title: Studying Evolution Using Artificial Life

Open to (expected background/level of study): Experience in computer science/ computer programming is required. Knowledge of C++ is particularly beneficial because Avida is written in this language.

Project description: During the past couple of decades computer scientists have created software platforms that mimic the process of biological evolution within computers. For example, in these platforms the genomes of individual organisms are represented by sequences of programming instructions, and each such "organism" produces offspring by replicating its genome (i.e., by replicating its programming instructions) into a new individual. Mutations also occur during this replication, however, leading to variation among individuals. Some of these individuals thereby turn out, by chance, to be better than others at replicating and so the composition of the population of these "digital organisms" evolves over time, with some programs outcompeting others. A beautiful example of this type of artificial life platform is Avida (http://avida.devosoft.org). In this project we will use Avida to address questions in evolutionary biology, first by learning how it currently works, and then by modifying its code in various ways.

Student role: To explore the use of Avida first by simply playing around with its current capabilities and then digging into the code for this platform and modifying it in new ways.

Supervisor: Felicia Magpantay

Project Title: Noisy data and Kalman-type filters

Project description: Kalman filter approaches are used in many applications (robotics, control, meteorology, biological modeling) to infer parameters and underlying states from noisy observations. If dynamics are linear and all the noise is Gaussian, the basic Kalman filter (KF) is the best linear estimator. The Ensemble Kalman filter (EnKF) and Ensemble Adjustment Kalman filter (EAKF) are algorithms based on KF that have been extended to work for nonlinear dynamics. These methods have been applied to high-dimensional problems for which more exact, full-information methods (e.g., sequential Monte Carlo approaches) are computationally infeasible. This will be joint work with Aaron King (University of Michigan).

Student role: The student will conduct a theoretical study of KF, EnKF, and EAKF algorithms and test their usefulness in fitting some simple models to data. The goal is to obtain a good understanding of when these methods are appropriate to be used and how well they can do in some non-Gaussian situations where KF methods are formally inappropriate but may still be useful as approximations. This will help us understand if and how such methods can contribute to full-information inference.

Open to (expected background/level of study): Undergraduate students applying to work on this project should have a theoretical probability background (MTHE/STAT 351 or equivalent) and have some basic familiarity with R.

Supervisor: Andrew Lewis

Project Title: Control of linear systems with constraints using the Baker-Campbell-Hausdorff formula

Open to (expected background/level of study): Some background in control theory, good mathematical background, some programming ability.

Project description: Controllability is a classical problem in control theory, and the controllability of linear systems with no control constraints has been well-known since the 1960's. In this project, the local controllability of linear systems is to be studies in the presence of constraints on the controls, e.g., controls take values in some compact set. The problem to be studied here is part of a larger project in controllability, and the subproblem in linear control theory is intended to gain some insight into how the mechanisms of controllability can be understood using methods from free algebra, such as the Baker-Campbell-Hausdorff formula. The final objective of the project is to prove or disprove a conjecture about the linear controllability with control constraints.

Student role: The project will begin with an exploration of the controllability problem, and of the machinery present in the conjecture that will be explored. After the background knowledge has been obtained, simple problems will be explored to flesh out the viability of the conjecture. If the conjecture survives the exploration phase, an assault on the proof will be mounted. If the conjecture does not survive the exploration phase, alternative characterisations will be explored.

Supervisor: Fady Alajaji

Project Title: Causal Coding and Stochastic Control in Two-Way Channels

Open to (expected background/level of study): Students at the end of their third or fourth year of studies.

Project description: The two-way channel (TWC), first introduced by Shannon in 1961, is a channel in which each user transmits and receives data simultaneously. It is a very important model as it represents the mode of communication in today's data communications systems. Yet the exact characterization of its capacity region, which is the set of all rate tuples for which the users can reliably send information to each other, remains an open problem. Indeed, even in the case of two users, the capacity region of the TWC is still not known in general. The difficulty mainly emanates from the fact that in TWCs, the encoder of each user needs to interactively adapt the current input to its own message and to all previously received signals. Such interactive coding introduces correlation between inputs of different users, making the analysis of the capacity region complicated. The capacity region is however known for a few TWC models in which the optimal encoding strategy of each user does not adapt to the previously received outputs. In this case, where such encoding "adaptation" is useless, the capacity region has been successfully derived; this holds for TWCs satisfying certain symmetry properties including the discrete additive noise and the Gaussian TWCs.

This project will explore stochastic control causal coding strategies for the reliable transmission of (discrete and analog valued) information sources over the TWC, starting with two-channel models for which the capacity region is exactly known (such as the models mentioned above). The derivation of optimal causal encoding/decoding functions under different distortion criteria will be examined. In particular, one objective is to elucidate system stochastic conditions under which linear coding strategies are optimal.

Student role: The student will be involved in all aspects of the project, including a thorough literature review and a meticulous understanding of the state-of-the-art results, mathematical derivations, performance analysis, performing numerical simulations and writing a detailed research report.

Supervisor: Daniel Offin

Project Title: The trapezoidal four body problem

Open to (expected background/level of study): 3rd or 4th year students who have taken differential equations and analysis courses and who have an interest in mathematical aspects of classical mechanics. Some computer background and in particular some ability with graphics for dynamical systems and differential equations.

Project description: I am interested to investigate the case of stability for families of homographic solutions in the trapezoidal four body problem. Some background preparation for this problem will be necessary, and then some computer simulations of periodic solutions and their stability type will be investigated. This a low dimensional dynamical system, governed by a set of Newtons' equations. The dynamics of such system is in general chaotic, but with some regularity which we try to investigate.

Student role:

Supervisor: Charles Paquette and Greg Smith

Project Title: Quivers and global dimension

Open to (expected background/level of study):

Project description: A (finite-dimensional) algebra is a vector space equipped with a bilinear product. The algebra of all (n×n)-matrices with matrix multiplication is a classic example. Combinatorial methods allow one to associated a directed graph (aka quiver) to any finite-dimensional algebra. Using this approach, we can encode the algebra in a square matrix over the integers, called the Cartan matrix. Conjecturally, the determinant of this matrix reflects the homological properties of the underlying algebra. More precisely, the complexity of an algebra can be measured by a numerical invariant called its global dimension, and the Cartan determinant conjecture states that, for every finite-dimensional algebra of finite global dimension, the determinant of its Cartan matrix equals one. In this summer project, we will develop the necessary background to understand this conjecture and attack it from several different perspective.

Student role: The undergraduate students will be involved in all aspects of the research project. This includes, but is not limited to, generating examples, formulating and testing conjectures, developing computer experiments, exploring the research literature, writing up proofs, and making presentations.

Supervisor: Thomas Barthelme and Francesco Cellarosi

Project Title: Spectral Hilbert geometry

Open to (expected background/level of study): Required courses: MATH/MTHE 281, MATH 210; recommended courses: MATH 310, MATH 328

Project description: A common and deep question in geometry is to understand when one can determine a metric uniquely from some spectral data ("Can one hear the shape of a drum?").

A famous result of Sunada states that there are many hyperbolic surfaces that have the same length spectrum (the set of length of closed geodesics counted with multiplicity). In this project, we aim to adapt this construction to surfaces endowed with Hilbert metrics.

Student role: In the first part of the project, the student(s) will learn some basics of hyperbolic and Hilbert geometry. In the second part, the student(s) will adapt Buser's version of Sunada construction of the surfaces from the hyperbolic to the Hilbert case. (see sections 11.5 and 11.6 of Buser "Geometry and spectra of compact Riemann surfaces")

Supervisor: Mike Roth

Project Title: Convex bodies, diophantine approximation, and toric surfaces

Open to (expected background/level of study): 2nd year math at least (including Math 210) + mathematical maturity. Having taken Math 413 a plus (but not necessary). Ideally, two students would work on the project together.

Project description: The project will try and understand recent results about diophantine approximation and blowups of P^2 in the setting of toric surfaces, with the goal of trying to prove a kind of 'Schmidt subspace theorem' for torus fixed divisors.

Student role: The student (or students) in the project will first learn about Diophantine approximation, a topic in number theory with applications to number theory and geometry, learn about toric surfaces, and their connection to convex polytopes, learn results connected with diophantine approximation, investigate the 'volume function' of a divisor on a toric surfaces, and apply these in an effort to prove a kind of 'Schmidt subspace theorem' for torus fixed divisors.

Supervisor: David Wehlau

Project Title: Symmetric Functions and Regular Sequences

Open to (expected background/level of study):

Project description: This project concerns symmetric functions, group actions and polynomial rings. We will concentrate on the ring of polynomials in 3 variables over the complex numbers C[x,y,z] and its natural action under the symmetric group on 3 letters. The goal is to study the triples of polynomials f,g, and h which are invariant under this action and which also have only a single common zero. This project offers an opportunity to develop familiarity with some important concepts in modern algebra by using them in a practical application.

Student role: The successful student will conduct research on this problem in collaboration. In addition, he or she will learn a computer algebra system as computer experiments are very helpful.

Supervisor: Ram Murty

Project Title: Number Theory

Open to (expected background/level of study):

Project description: There are many Diophantine questions that can be related to the theory of elliptic curves and modular forms. The project requires a knowledge of both these areas along with some ability in analytic and algebraic number theory. The project involves applying what is now called the "modular method" to attack some old, unresolved Diophantine problems.
Progress will be measured through the weekly number theory seminar through lectures and discussions.
Student role:

Supervisor: Glen Takahara, David Thomson, Devon Lin with Emily Somerset

Project Title: Second Order Periodicities in Interplanetary Magnetic Field Data

Open to (expected background/level of study): Statistical and computing background are as follows:
1. The student should have some knowledge of statistical inference procedures, specifically hypothesis testing and estimation. Past exposure to time series analysis and spectrum estimation is desirable as these will be the main statistical areas of the project methodologies, but these can be learned during the project tenure.
2. Familiarity with the R statistical software is highly desirable. All the coding and computations will be done in R. Code for much of the analysis can be provided, but further coding by the student will almost certainly be required.

Project description:
Background: A recent paper by Fossat, et al. ("Asymptotic g modes: Evidence for a rapid rotation of the solar core", Astronomy and Astrophysics, August 2017) has been able to accurately measure the rotation of the solar core, for the first time in 40 years, based on the detection and measurement of gravity waves, also known as g-modes, which have themselves been only theorized to exist in the sun in the field of helioseismology. The data that was analyzed is from the European Space Agency's and NASA's Solar and Heliospheric Observatory spacecraft, or SOHO, collected by the Global Oscillations at Low Frequencies, or GOLF, instrument. According to SOHO project scientist Bernhard Fleck, the accurate measurement of the rotation of the solar core "is certainly the biggest result of SOHO in the last decade, and one of SOHO's all-time top discoveries". A press release from NASA at https://www.nasa.gov/feature/goddard/2017/esa-nasa-s-soho-reveals-rapidly-rotating-solar-core and a short news article at https://www.sciencealert.com/scientists-just-revealed-a-surprising-secret-about-the-sun-s-hidden-core give a lay description of the findings in this paper. Of special interest for this project is the finding that the mean solar core rotation rate was approximately 1.28 microhertz in frequency, which is mapped from measured g-mode splittings. A recent Master's thesis by Emily Somerset at Queen's ("Multitaper methods for cyclostationary feature detection in time series data: application to ACE interplanetary magnetic field data", December 2017) found structure at this very same frequency in the sun's magnetic field, using data from the Advanced Composition Explorer (ACE) spacecraft, which was launched by NASA in 1997 and whose mission is to sample low energy particles originating from the sun. The oscillations at 1.28 microhertz (roughly a 9 day period) were found in the second order properties of the ACE IMF data, most prominently during periods of low solar activity. It is an interesting question whether this oscillation in the solar magnetic field is connected to the gravity wave oscillation in the solar core and whether these magnetic field oscillations can provide a further signature for the detection of g-modes.

Goals: The project would entail reading the Fossat, et al. paper and Emily's thesis to get a background on the methods and outcomes of these documents. Then one or more of the following items are the remaining project goals, depending on time availability:

1. Reproduce the main results in the Fossat, et al. paper and Emily's thesis. The data to reproduce these results will be provided.
2. Take the segment of ACE data that shows the strongest cyclostationarity. Learn how to standardize the spectrum as done in the appendix of Thomson, et al., "Interplanetary magnetic field: statistical properties and discrete modes", Journal of Geophysical Research, August 2001. After standardizing, set the significance level at 99%. Assuming the lines surpassing that line are modal lines, see if these are lines are often separated by roughly 1.28 microhertz.
3. Search the helioseismology literature for a possible theoretical connection between gravity waves in the solar core and second order periodicity in the interplanetary magnetic field.

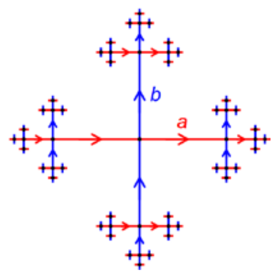
Student role:

Supervisor: Jamie Mingo

Project Title: The Fluctuations of the Kesten-McKay Law

Open to (expected background/level of study): Prerequisites are a course in real analysis (e.g. math 281), a course in complex analysis, and a course in probability theory.

Project description:



The Kesten-McKay law is a probability distribution counting random walks on the free group on d generators as well as the eigenvalue distribution of the adjacency matrix of a random d-regular graph, here d is a positive integer.

Using a random matrix model of the free group we can compute the fluctuations of this law. The goal of the project is to find the density that yields the fluctuation moments.

Student role:

The following NSERC USRA project is being offered by Professor Dongsheng Tu, Department of Public Health Sciences. Applicants should apply directly to Professor Tu.

Project Title: Statistical Methods and Analysis of Data from Cancer Clinical Trials
Open to (expected background/level of study): Domestic students at the end of their third or fourth year of studies.
Project description: This project will involve evaluation of statistical methods and the analysis of data from cancer clinical trials conducted by Canadian Cancer Trials Group located at Queen's University Cancer Research Institute.
Student role: The successful student will use SAS or R to write programs for the data analysis and computer simulations.

The following project is being offered by Dr. Mohan Chaudhry of the Department of Mathematics and Computer Science at the Royal Military College of Canada. There are up to three positions available for this project including one USRA position and one part-time or full-time position for the coming summer term and another USRA position for the 2019 winter term.

Students interested in applying to work on this project should contact Dr. Chaudhry directly by email or phone (613-541-6000/6460):

Project Title: Inverting transforms that arise in the study of Markov models

Project description: Many of the analytic solutions in queueing and other stochastic processes are derived in various transforms such as probability generating functions and Laplace transforms. The problems become more complicated if there are unknowns in the transforms. Several complicated algorithms/methods have been proposed to invert such transforms. We have developed a software program which inverts such transforms using the roots of high degree polynomials and transcendental functions. Our method of inverting such transforms is much more efficient and fast when compared with other methods.

Student role: The student's role will be to invert such transforms using mathematical tools such as MAPLE/MATLAB or MATHEMATICA and QROOT, a software developed by us as well as do some mathematical typing.