Math 328
Real Analysis
Winter 2007

Textbook: *Elementary Classical Analysis* (second edition) by J.E. Marsden and M.J. Hoffman.

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Office hours: Tuesdays, 11:30.

Times and Place:

Times: slot 3 (Mon. 10:30, Wed. 9:30, Fri. 8:30)
Place: Jeffery Hall, room 319.

Assignments:

Assignment 1
Assignment 2
Assignment 3
Assignment 4
Assignment 5 (due April 6)

Handouts:

Cardinality and Schroeder-Bernstein theorem
Completeness of the Hausdorff metric

Course outline: We will cover roughly chapters 0--7 of the textbook. The main concepts to be studied are:

Sets and functions between sets: Infinite sets, cardinality:

Topology, topological

spaces: Open and closed sets, continuity, compactness, connectedness;

Metric spaces: Convergence, completeness, normed spaces, scalar product spaces, contraction mapping principle and applications.

Derivative as a linear map: Chain rule, linear approximations, Taylor approximations, polynomial approximations;
inverse function theorem, implicit function theorem with applications.

In the proposed readings I do not include the page references for the proofs. It is understood that, when the assigned reading includes a theorem, the proof should be studied as well.

Week 1:
*From the Introductory chapter: Sets and functions between sets, infinite sets and cardinality, countable sets, Schroeder-Bernstein Theorem (see handout), proof that rationals are countable, and the interval [0,1] is not.

Week 2:
*From Chapter 1 in the textbook: Review the main properties of rational and real numbers: completeness of the real line, density of rationals in the set of reals, sup and inf, Bolzano-Weierstrass Theorem for the real line, cluster points of a sequence;

Reading for week 2:
*Read pages 31-35 as a review.
*Read pages 35-40 as a review. However, in the book the material is presented as a part of axiomatic treatment of the real line:
the set of real numbers is characterized as a "complete ordered field". We shall not be interested in the axiomatic aspects of this discussion. Therefore pages 35-40 should be seen as a review of the properties of the real line. By exception, it is not necessary to read the proofs associated to these pages.
*Read pages 44-64. Most of this material will be familiar.

Week 3:
*From Chapter 1: metric spaces, normed linear spaces and inner product spaces with examples, Cauchy Schwartz inequality.
*From Chapter 2: The first part of this material will be treated in a greater generality than it is done in the book (see the boxed comment at the bottom of page 106). In the lectures we shall define the concept of a topological space and give examples of topological spaces. We shall show that metric spaces are examples of topological spaces. We shall discuss open sets, interior of a set, closed sets, accumulation points, closure of a set, boundary of a set.

Reading for week 3:
*Read pages 64-69. The concepts presented there, especially that of a metric space, may be new. In the lectures we shall discuss some examples not presented in the book.
*Read pages 103-119.

Week 4:
*From Chapter 2: Continue discussing open sets, interior of a set, closed sets, accumulation points, closure of a set, boundary of a set (in the context of topological spaces). After that, and only in the context of metric spaces: sequences, convergence, Cauchy sequences, completeness, brief discussion of series (on normed linear spaces), Banach spaces, absolute convergence, completeness of the space of continuous functions on a closed interval (uniform convergence).

Reading for week 4:
*Read pages 120-129. A lot of this is a review of material learned in the second year.

Week 5:
* From Chapter 3: compact sets, sequential compactness, Bolzano-Weierstrass theorem.

Reading for week 5:
*Read pages 151-156. This material is new and important. The proof of the Bolzano-Weierstrass theorem is long and difficult.

Week 6:
* From Chapter 3: nested set property, path-connected sets, connected sets, limit.
* From Chapter 4: four definitions of continuity (Theorem 4.1.1), composition of continuous functions, continuous images of compact and connectd sets, Maximum-Minimum Theorem, Intermediate Value Theorem.

Reading for week 6:
*Read pages 157--164;
*Read pages 177--193.

Week 7:
* From Chapter 4: uniform continuity and the Unifirm Continuity theorem;
* From Chapter 5: uniform convergence, continiuty of the uniform limit of a sequence or series of functions, Weierstrass M-test, The Cauchy criterion and uniform Cauchy sequences; the space of bounded continuous functions and the space of continuous functions on a compact as metric spaces, with completeness if the metric space is complete.

Reading for week 7:
*Read pages 194--195;
*Read pages 237--246.
*Read pages 268--272.

Week 8:
* From Chapter 5: Integration and differentiation of series; Arcela-Ascoli theorem.

Reading for week 8:
*Read pages 247-253;
*Read pages 272-274.

Week 9:
* From Chapter 5: The Contraction Mapping Principle with applications: existence and uniqueness of solutions for ordinary differential equations, integral equations. Stone-Weierstrass theorem.

Reading for week 9:
*Read pages 275-285.

Week 10:
* From Chapter 6: Review differentiable mappings between Euclidean spaces: derivative as a linear mapping, techniques of calculation of the derivative, Chain Rule, the Mean Value Theorem, higher order derivatives as multilinear mappings, Taylor's theorem.

Reading for week 9:
*Read pages 327-362.

Weeks 11-12:
* From Chapter 7: Inverse Function theorem and Implicit Function theorem with applications: Domain Straightening theorem, Range Straightening theorem, Morse Lemma (if we have time :)).

Reading for weeks 11-12:
*Read pages 391-406.

Evaluation:	Homework	70%
	Final examination	30% .