Mathematics 010* "Fundamental Concepts in Elementary Mathematics for Teachers"

Course Description

The contents of this course alternate on a two-year cycle, to accommodate the ***StepAhead*** enrichment program associated with the course. As part of the course requirement, students go out in pairs to provide a ten-week, one hour per week, mathematics enrichment program for students in grades 7 and 8 in local schools. In odd numbered years the course focuses on Numbers and Number patterns, to support the ***StepAhead*** **Numbers** enrichment program. In even years the focus is on Geometry to support ***StepAhead*** **Geometry**.

In each case, the course material is presented in a form that should be applicable in a grade 7 or grade 8 class. This has two purposes: It helps students think of the mathematics in a form that is useful for their enrichment teaching; it gets at the (non-computational) heart of the mathematics.

In particular, the course uses almost no algebra (to avoid a formulaic approach) and lots of problems, puzzles and games to contextualize the mathematics and to present it in a form that can be adapted to the middle school classroom.

At the end of the course we often spend some time discussing even more elementary material more directly related to the classroom, such as teaching fractions or negative numbers, but this content varies quite a bit from one year to the next and in any case takes up a rather small number of classes.

Assessment in the course is as follows:

15% for class participation
15% for written reflections on enrichment classes
55% for ten assignment problems
15% for a final exam

In addition, the enrichment teaching experience provides a very effective form of self-assessment and constitutes strong motivation for deep learning.

The syllabi for the two versions of the course are given below.

The Numbers version of Math 010* (odd years)

- **Number patterns and pattern rules**
- The idea of infinity (Hilbert's infinite motel);
- Functions and how to read a formula;
- Explanations (proofs) for cases when different proofs produce the same sequence;
- Growth rates of sequences.

- **Prime factorization of a number**
- Divisibility tricks for divisibility by 2, 3, 4, 5, 9;
- Properties of numbers that are a consequence of their prime factorizations;
- A number trick that depends on prime factors;
- How many primes are there?
- The search for large primes;
- Using prime factorizations to obtain greatest common divisors;
- Using prime factorizations to obtain least common multiples.

- **Fractions and decimals**
- Converting a fraction into a decimal;
- Converting a decimal into a fraction (includes repeating decimals);
- Irrational numbers and non-repeating decimals;
- Proof that the square root of 2 is irrational;
- Discussion of real numbers as points on a line ("magic meter stick");
- Is 0.999... equal to 1 or not?

- Modular Arithmetic(the arithmetic of remainders)
- Problems illustrating the role of this arithmetic;
- Addition and Multiplication tables modulo 2, 3, 5, ...;
- Explanation why the divisibility tricks for 3 and 9 work the way they do;
- A divisibility trick for 11.

- **Counting**
- Problems involving coins and dice.

- **Probability**
- Discussion of equally likely outcomes;
- A variety of problems and examples.

The Geometry version of Math 010* (even years)

- **The theorem of Pythagoras**, introduced using a problem in 3d Geometry;
- A discussion of the Theorem of Pythagoras: What does it claim? What does it do? How do we know it is true? How would you introduce it to students in grades 7 or 8?
- A proof of the theorem, appropriate for discussion in the enrichment program. This includes a discussion of how proof differs from inductive evidence.
- Many problems involving Pythagoras' Theorem:
    - The spider and the fly (distances in a rectangular room);
    - The crochet needle stuck in a vacuum cleaner tube;
    - Packaging rolls of candy.

- **Similarity transformations**: An examination of the properties (particularly ratios of length measurements that are preserved when we zoom in or out to a figure.
- The problem of "Archimedes' Pool": fitting the largest possible equilateral triangle into a standard sheet of paper.
- Fitting the largest possible octagon on a standard sheet of paper.

- **The Cube, octahedron and tetrahedron.**
- Examination of profiles of these regular polyhedra;
- Examination of duality between them;
- The smallest possible cube to fit around a given tetrahedron.

- **The Euler number** for General Polyhedra
- Discussion of topological shape (without pushing the technical language: surfaces are "sphere-like" or "donut-like" etc.);
- Examination of the Euler number as a way to distinguish surface;
- A proof that the Euler number for a "sphere-like" surface is always 2;
- Definition of regular, as in regular polyhedron;
- Search for (and discovery of) other regular polyhedra (the Platonic solids);
- Templates (nets) for these solids.

- **Area** as a way of counting how many unit squares fit into a figure;
- The length and width of the largest rectangular vegetable garden surrounded by 16 meters of fencing;
- The same problem when it is built against an existing fence;
- The largest volume of an open-top box that can be folded from a 20cm x 20cm sheet of paper;
- Graphs and formulas associated with these problems;
- Growth rates of areas and volumes under similarities;
- The areas of parallelograms and triangles;
- The effect of shears on area;
- Describing and finding the area of the shadow cast by a rectangular table supporting a table lamp;
- Describing and finding the area of the shadow cast by a rectangular fence and a single light source.

- **Area and perimeter of a circle.**
- Where does the number π come from?
- Estimates of π using Archimedes' technique;

- Why do both area and perimeter involve the same number π?
- Various applications;
- The toilet paper problem: finding the thickness of toilet paper.