Math 338 - Topics in Applied Mathematics

In this course we are interested in generating and understanding various solutions to certain "classical" partial differential equations such as the heat, wave, and Laplace equations with various boundary conditions. These classical equations arise from modelling simple phyiscal problems. For example the wave equation can be used to model a vibrating string with fixed end points or in 2-dimensions the vibrations of a drum. The actual mechanics, a method called "separation of variables", for solving such equations are in fact deceivingly easy whereas the underlying mathematics is not. The method of "separation of variables" employs orthogonal (Fourier, Bessel, eigenfunction) series expansions to construct a formal power series solution and it is here in which the mathematical subtleties are contained.

- **Prerequisite**: MATH 227 or MATH 280, MATH 226 or MATH 237 or MATH 232, or permission of the instructor
- **Instructor**: David R. Tyner
- **Web**: http://www.mast.queensu.ca/ ∼ math338/
- **Required text**: Partial Differential Equations and Boundary Value Problems
  with Fourier Series,
  Nakhle H. Asmar, 2nd Edition
  Publisher: Prentice Hall
  ISBN: 0-13-148096-0
- **Evaluation**: Final Examination 60%
  Midterm Examination 20%
  Assignments 20%
- **Outline**:
  1. Course introduction, linear algebra review, basic functional analysis (function spaces, linearly independence, inner product and normed spaces, orthonormal families)
  2. Periodic functions, Fourier series, Fourier coefficients, periodic extensions
  3. Pointwise and uniform convergence, piecewise continuous and smooth functions, convergence criterion, operations on Fourier series
  4. The Heat equation (1-dimenional): derivation, steady state solution, full solution via seperation of variables, mixed boundary conditions, understanding the solution
  5. The Wave equation (1-dimensional): derivation, fixed end point solution, d'Alembert's solution
  6. The Laplace equation (2-dimensional in cartesian coordinates): definition, solution to Dirichlet boundary conditions
  7. Sturm-Liouville problems: definitions, properties of eigenfunctions for both regular and singular problems, eigenfunction expansions
  8. Higher dimensional BVPs in polar and cylinderical coordinates and Laplace's equation on a disk
  9. Bessel and modified Bessel equations: the Gamma function, power series solutions, properties of the solutions, Bessel series expansions
  10. Vibration of a circular drum: radially symmetric solution, general solutions