MATH 386*

Advanced Elementary Number Systems - Fall 2008

This course is suitable for all students in a MATH Honours programme. It is one of the courses in the department's Teaching Focus.

Instructor: Prof. M. Orzech, 210 Jeffery Hall, 533-2436, orzechm@mast.queensu.ca

Recommended background: One of 210*, 211, 212*, 281*.

Evaluation scheme: 30% for homework; 20% for midterm test (Tuesday of Week 6), 25% for report (and possibly presentation) on group-work term project; 25% for final exam.

Homework: About twelve problems will be assigned over the term. You will have at least one week to work on each problem set.

Course description:

Math 386 is about "number systems": integers, rationals, reals and extensions. We examine some fundamental properties of real numbers and related number systems, and the connection of these properties to calculus and to other mathematical topics encountered earlier by students. We discuss usual and unusual representations of numbers (e.g., in different bases, as sums of unit fractions, as continued fractions), which properties depend on representation, and why. The course touches in a "non-abstract" way on ideas from analysis. For example, convergence, upper bounds, and Cauchy sequence are introduced (reviewed for some students) in considering foundational properties of numbers that students often find difficult to explain or illustrate or connect to more elementary things we know. One goal is to probe the basis for our sense of familiarity with the real numbers and their arithmetic, including our understanding that the set of real numbers is complete. This will lead to the realization (echoing a 19^{th} century phenomenon) that using familiar convergence properties of real sequences can seem like "circular reasoning" until one explains how the real numbers arise from the rationals. The construction of real numbers as Dedekind cuts or as Cauchy sequences on the rationals will be outlined, with some exercises, but not complete proof. In past years there has been time for a segment on non-standard models of the reals, and for explaining Liouville's construction of transcendental numbers.

Classes will feature a variety of activities, including lectures, class exercises and discussion, and student presentations (which may be with a partner, and for which guidance and preparation support will be available). In addition, students will work in a small group on a substantial independent study of one of several chosen or assigned topics. This work will be written up as a report, and insofar as class size permits, each group will do a class presentation based on its work.